Thèse pour le Grade de Doctorat en Sciences Économiques

Transition d’Emploi à Emploi et Dispersion des Salaires: Approches Théoriques et Empiriques

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General Introduction

Equilibrium search theory emphasizes the importance of labor market frictions for the understanding of labor market outcomes like wages and unemployment (see Mortensen[1998], Pissarides[2000], Mortensen[2002a]). Central to the theory is the notion of market frictions. It takes time and other resources for a worker to find a job, especially a good job at a good wage, and for a firm to fill a vacancy. As a result of market frictions and Bertrand competition among the firm, pure wage dispersion is a robust equilibrium outcome provided that workers search on-the-job (see Burdett-Mortensen[1998]).

Nevertheless, the pure wage dispersion model of Burdett-Mortensen[1998] has problems fitting empirical wage data. In particular, the model implies that the equilibrium offer and earnings densities increase monotonously across their common support in apparent conflict with the uni-modal shape typically observed. So in order to match the observed wage dispersion, recent studies have stressed the essential importance of firm heterogeneity (measured and unmeasured) in interaction with market frictions.

Let’s inspect this theory along another line. Equilibrium search-matching theory typically views labor markets as job ladders. Workers search for good matches while unemployed and while employed. Employment with a particular firm ends either when the job is destroyed or when a worker finds a better opportunity. This simple structure yields several strong predictions. Incomes increase as workers move from lower paying to higher paying jobs, occasionally interrupted by spells of unemployment as some jobs are destroyed and the worker has to start over again on the lower rungs of the ladder. Therefore, the probability of job-to-job transition always decreases with the wage of the worker. Of course, this job ladder view of the labor market is necessarily too simplistic in some dimensions. Detecting what those dimensions are and to
what extent, and understanding why the job ladder paradigm fails are all issues of great importance.

The outline of the thesis is as follows. In the first chapter, we survey the recent developments of equilibrium search theory and focus on a fundamental question of labor economics: why are workers paid differently? We first present the empirical studies that investigate the determinants of wage differentials and wage variation. We then explain wage dispersion along four lines: pure wage dispersion, wage dispersion linked with exogenous or endogenous heterogeneity, and intra-firm wage dispersion and wage growth.

In chapter 2, we proceed to empirical analysis of equilibrium search model along two directions: not only wage dispersion but also job-to-job transitions. Using French panel data (French Labor Survey 1990-1999), we decompose the total flows of job-to-job transition into flows of promotion and flows of external mobility. Formally, we consider three types of job-to-job mobility: external mobility (without promotion), external promotion and internal promotion. By promotion, we mean a raise of occupational category of the employee in sense of Social Professional Category (SPC). By external mobility, we mean a job-to-job mobility via a change of firm (made with an episode of unemployment of one month or less). Importantly, an internal promotion should also be considered as a job-to-job transition because it concerns a change of job (see Lazear-Oyer[2004]).

We find that, as revealed in most studies, the shape of wage density is systematically uni-modal and of log-normal type. We also find that there exists an ambiguous relation between the frequency of job-to-job transition and the wage, typically first decreasing then increasing. We show that this ambiguity refers to two particular types of professional mobility: (1) a job-to-job mobility that results in a wage decline, and (2) a promotion that results in a raise of occupational category (SPC). We then structurally estimate the Burdett-Mortensen[1998] model while allowing for a continuous productivity distribution, and compare the predictions of the model with the actual data. All the above empirical findings indicate that an extension of the canonical equilibrium search model is necessary to provide more realistic predictions to wage and job trajectories of individuals in the labor market.
In chapter 3, we develop the job search theory to give an alternative explanation of equilibrium wage distribution. We reinspect the role of endogenous search effort in a pure environment where all workers and firms are respectively identical. We distinguish two types of job: low-productivity jobs and high-productivity jobs. In low-productivity jobs, the wage is posted by the firm. In particular, workers in low productivity jobs are allowed to search on-the-job not only for wage increases, but also for opportunities of promotion. Once a worker accedes to a high productivity job, it is assumed that the wage is negotiated according to a sharing rule. We also assume that employees in high-productivity jobs segment could only be recruited from low-productivity jobs segment, they stop searching on-the-job and face no more firing risk. We show that, depending on the non-separability of preferences between consumption and leisure, the model allows to capture that wage distribution within either jobs segment is bell-shaped.

In chapter 4, we turn to investigate the impact of firm behavior to market wage dispersion and, in particular, job-to-job transitions. We also distinguish two jobs segments: low-productivity jobs segment and high-productivity jobs segment. Firms are allowed to make human capital investment in low-productivity jobs segment. On the other side, firms pay a fixed cost of training for each newly-created job in high-productivity jobs segment. We then structurally estimate the model using the simulated method of moments (SMM). The estimation results show that the model gives, in general, a good fit to the actual wage distribution. Importantly, the model permits to replicate a positive relation between probability of promotion and wage, and a U-shaped relation between probability of job-to-job transition and wage. These characteristics coincide well with the French data.
GENERAL INTRODUCTION
# Chapter 1

**Explanations of wage dispersion: why are workers paid differently?**

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1.1 Introduction

Generally speaking, job search theory permits to provide a richer framework for labor market analysis than that provided by the frictionless competitive model. While the usual paradigm of supply and demand in a frictionless labor market is useful for discussing some issues, many important questions are not easily addressed with this approach. For example, how can we simultaneously have unemployed workers and unfilled vacancies? What factors determine the aggregate unemployment and vacancy rates? What determines the lengths of employment and unemployment spells? How can apparently homogeneous workers in similar jobs end up earning different wages? What are the trade-offs faced by firms in paying different wages? What determines the efficient amount of turnover? And many others.

From its inception, search theory has provided a rigorous yet tractable framework that can be used to address these and related questions. Central to the approach is the notion that trading frictions are important. It takes time and other resources for a worker to find a job, especially a good job at a good wage, and for a firm to fill a vacancy. There is simply no such thing as a centralized market where buyers and sellers of labor meet and trade at a single price, as assumed in classical equilibrium theory. Of course, economic models do not have to be realistic to be useful, and the supply-and-demand paradigm is obviously useful for studying many issues in labor economics. But it is equally clear that the simple supply-and-demand approach is not suited for discussing questions such as those raised in the previous paragraph.

In this chapter, we survey the literature on the recent developments in models of search in the labor market. For a good understanding of the relevance of this literature, it is useful to start with an examination of some earlier contributions in the search literature, the so-called partial job search models (see e.g. McCall[1965], Stigler[1961], Phelps and Mortensen[1970]). These models deal with job search behavior of unemployed or employed workers. The worker’s optimal strategy is derived for a given distribution of wage offers and for given job offer arrival rates. The literature on partial job search models has made considerable progress in explaining supply-side behavior,
unemployment durations, and job durations of workers looking for a (new) job.

However in this partial approach, the distribution of wage offers is assumed to be fixed and is treated as exogenous. The most serious drawback of this is that the model is not able to deal with changes in the employers’ behavior in response to policy changes or other structural changes. Furthermore, one major problem with structural inference of partial models has been that the wage offer distribution can not be identified from the type of data that are usually collected. If all job seekers have a common reservation wage and face a common wage offer distribution, then data on accepted wages amount to a sample from a truncated distribution where the reservation wage is the lower boundary of the support. One cannot identify (recover) the complete wage offer distribution from this truncated distribution. In particular, one cannot identify the probability mass below the reservation wage. This is a serious problem, because that probability mass is the probability that a job seeker rejects a wage offer. To deal with it, a common procedure in applications is to restrict the wage offer distribution to a parametric family of densities that is recoverable, so the un-truncated density can be uniquely recovered from a truncated density. Usually, a family is chosen that is known to be able to give a good representation of wage distribution (e.g. the lognormal family). Nevertheless, the underlying distributional assumptions are un-testable (see Flinn-Heckman [1982], Van den Berg [1994, 1998]).

Another problem with partial models concerns the assumption that wage offers are dispersed. It is well known that non-cooperative price posting under perfect information generates a Bertrand equilibrium, one in which all charge a common competitive equilibrium price, even when the number of competitors is small. Diamond [1971] was the first to solve a fully consistent equilibrium version of the price posting game under imperfect information about offers. He finds that only the monopsony wage is offered in equilibrium even when the number of competitors is large. The argument is simple: the optimal strategy with sequential search is a reservation wage strategy. For the employers it is suboptimal to offer a wage that is higher than the common reservation wage, as a higher wage means that profits per worker are lower while the steady state number of workers is not higher. Hence, the
equilibrium wage offer distribution is concentrated at this reservation wage. Finally, this common reservation wage must be equal to the lowest wage that is acceptable to the unemployed, i.e., their value of leisure. Indeed, Diamond’s unique solution to a wage posting game when workers and employers are identical yields a market equilibrium in which no workers are willing to participate if the cost of gathering wage information is strictly positive. This result is known as the Diamond paradox. Putting it differently, the Diamond paradox states, from the theoretical side, that the simple sequential search model, in which identical unemployed workers who face identical firms accept/reject wage offers one at a time, is not consistent with a disperse wage equilibrium; and challenges any empirical study with search models to address the issue of the origin of wage dispersion.

In response to all this, a literature called equilibrium search models has emerged. From the publication of the Diamond[1971] paper to the end of the last century, the contributions of Butters[1977], Burdett-Judd[1983], Albrecht-Axell[1984], Mortensen-Neumann[1988], Mortensen[1990], Burdett[1990], Burdett-Mortensen[1998] and Mortensen[1998], have provided insights into how a dispersed wage equilibrium can exist, or more precisely, how difficult it is to generate dispersed wages as an equilibrium phenomena. These authors show that persistent wage differentials are consistent with strategic wage formation in an environment characterized by market frictions with or without observable heterogeneity across workers and jobs. In equilibrium search models, the wage offer distribution is endogenous. It results from optimal wage setting by firms that take account of the behavior by job seekers and other firms. As a result, it is affected by all structural parameters, including policy parameters.

Most equilibrium search models share the common structure(see Burdett-Mortensen[1998]). Search is random, sequential and non-directed. Unemployed workers search for a job and, more importantly, employees search for a better job. Employer posts a wage conditional on the search behavior of workers and the wages offered by other firms. The wage posting approach is consistent with the idea that each employer is free to choose a particular wage policy, say to be either a “high-” or a “low-”wage firm. Given the wages offered
CHAPTER 1. EXPLANATION OF WAGE DISPERSION

by all others and the distribution of worker reservation wages, the labor force available to a specific employer evolves in response to the employer’s wage. The higher the wage the larger the steady-state labor force, because higher wage firms are more attractive to outsiders and retain insiders more readily. The resulting labor supply relation determines the profit of each employer conditional on the wage offered by other employers and the reservation wages demanded by workers. Facing the same trade off between wage and firm size, some firms choose the high wage even though profit generated per worker is lower, making up the difference in high volume. Consequently, wage dispersion exists in equilibrium even when workers are equally productive in all jobs.

We should also stress that, in addition to explain equilibrium wage dispersion, there have been other motivations for inference with equilibrium search models. In particular, equilibrium search models help to deal with a large number of interesting issues of labor economics\(^1\). Nevertheless, equilibrium search models are specially powerful in treating the following subjects:

- First, large numbers of workers flows between labor market states, as well as job creation and destruction flows, are reflected in activity spells typically found in panel data whose durations reflect the time spent searching for work, filling a vacancy, and working in a particular job. These movements are concealed in most existing models of employment that focus on stocks. The emphasis on mobility makes the equilibrium search models part of the so-called flows approach (see Blanchard and Diamond [1992]) to labor market analysis.

- Second, equilibrium search models allow for a variety of wage determination rules. Although the usual rule is wage posting, alternatives such as wage bargaining, monopoly union specification, insider-outsider or efficient wage theories can all be accommodated and studied within the framework. This merit makes the equilibrium search models a suitable workhorse to study the institutional structures of the labor market.

\(^1\)These include: labor force participation, unemployment and job duration, labor turnover, job creation and job destruction, institutional structures of the labor market, labor market and the macroeconomics, efficiency and design of policy, etc.
• Third, equilibrium search models provide a framework that allows for richer policy experiments. For example, because unemployment has an economic role in the flows framework, welfare statements about the effects of policy on unemployment and on the cost of unemployment experienced by those who bear it are possible. Also, the total effect of a policy can be decomposed into effects on unemployment duration and on unemployment incidence. As a consequence of this fact and the two-sided nature of the equilibrium search models, multiple channels of political influence arise.

In the following sections, we will survey the recent developments of equilibrium search theory and focus on a fundamental question of labor economics: why are workers paid differently? Before proceeding, the following guidelines should be established.

We first mention that search theory constitutes a very large branch of economics. In addition to labor markets, it has been applied in many areas in both micro and macroeconomics, including monetary theory, industrial organization, growth, public finance, and the economics of the marriage market\(^2\). All of these applications other than labor economics must be neglected to keep this survey pertinent.

Also we should point out that there are two quite different approaches to labor market analysis in the block “equilibrium search theories”, each with its own primary concerns. In the “matching approach\(^3\)”, the job creation flow is endogenously determined and depends on the numbers of unemployed workers and vacant jobs available. The goal of this approach is to explain worker and job flows and levels of unemployment. Also this approach proved successful in the modeling of the labor market in equilibrium business cycle models. In the “search approach”, search friction is regarded as exogenous and

\(^2\)Examples in monetary theory include Kiyotaki-Wright [1989, 1993], Trejos-Wright[1995]; examples in the marriage market include Mortensen[1988], Burdett-Coles[1997, 1999]; examples in industrial organization include Jovanovic-MacDonald[1994], Fishman-Rob[2000]; examples in finance include Duffie-Garleanu-Pedersen[2002], Weill[2004].

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simply the time required for workers to gather information about wage offers. The goal of this approach is to explain wage differentials, labor turnover and unemployment and job duration. This survey will focus on the “search approach” to labor market analysis in the following sections, whereas the “matching approach” is largely neglected.

This chapter is organized as following. We first present in section 1.2 empirical results on the study of wage differentials. The theory of pure wage dispersion (Burdett-Mortensen [1998]) is next presented in section 1.3. Because all firms and all workers are respectively identical ex ante, wage dispersion in this circumstance is called pure wage dispersion: any form of heterogeneity is not required to generate wage dispersion.

The Burdett-Mortensen [1998] model has problems fitting empirical wage data. In particular, the model predicts that the equilibrium wage earnings density increases monotonically across the support in apparent conflict with the single modal shape typically observed. So in order to match the observed wage dispersion we need some form of heterogeneity. By heterogeneity we mean that, for example, workers differ in ability or firms differ in the marginal productivity. Heterogeneity could either be exogenously introduced into the model as an assumption, or be generated as an endogenous outcome in market equilibrium. Sections 1.4 and 1.5 are then devoted to discussing the impact of firm or worker heterogeneity on equilibrium wage distribution by treating heterogeneity as exogenous and endogenous, respectively.

In all the models considered to this point, wages are invariant over the duration of a job match by assumption. Recent work relaxes this restriction for the purpose of explaining intra-firm wage dispersion, particularly that associated with job tenure. These studies are reviewed in section 1.6. Finally, the last section concludes the chapter.

1.2 Explanations of wage differentials: empirical evidences

A large variability in the wage earnings across individuals is typically observed in the data (see figure 1.1). Accounting for observable individual char-
1.2. EMPIRICAL EVIDENCES

Figure 1.1: Actual wage densities

Source: French labor survey (1990-1999)
characteristics helps not too much to explain observed wage differentials. Although numerous empirical studies that estimate different versions of wage equations verify that wage and workers characteristics—that one could view as indicators of labor productivity—are positively correlated, the explanatory power of observable individual characteristics is far from satisfactory. In particular, observable workers characteristics that are supposed to account for productivity differences typically explain no more than 30% of the variation in compensation across individuals, the other 70% of the variation remains unexplained by worker characteristics. These results motivate the question addressed in the literature: why are similar workers paid differently?

Some explanations argue that any available data set is incomplete in the sense that some relevant worker characteristics are not included. Isn’t it possible that the unexplained variation simply represents relevant unobserved individual characteristics? Other explanations predict that most of the variation is due to differential firm or industry compensation policies that do not follow the individual from job to job. Economists’ ability to distinguish among these explanations has been hampered by the lack of appropriate matched, longitudinal employer-employee data. For most E.U. countries, data of this type has only become available from the end of the last century. The key feature of such data is that individuals and employing firms are both identified and followed over time. Measured characteristics of the individual are collected at multiple points in time and measured characteristics of the employing firm are also measured at multiple points in time. For analysis of labor market, the matched employer-employee data is particularly useful in studying two subjects of interest. The first is the relative importance of person and firm variables in the determination of compensation. The second is the relative importance of individual mobility in relation to firm-specific employment adjustments.

In two related articles Abowd-Finer-Kramarz[1999] and Abowd-Kramarz-

\footnote{See Idson and Oi[1999], Mortensen[2002a].}

\footnote{See Abowd and Kramarz[1999a] for a general presentation of these linked employer-employee data. This paper surveys about 100 studies about data of this kind from more than 15 different countries. Virtually all of these papers have been written at the end of the 1990s.}
Margolis[1999] provided a basic statistical framework for decomposing inter-industry wage differentials and firm-size wage differentials into the sum of components due to individual heterogeneity (measured and unmeasured) and firm heterogeneity (measured and unmeasured). Both Abowd-Kramarz-Margolis[1999] and Abowd-Finer-Kramarz[1999] used statistical approximations to estimate the decomposition of wage differentials into individual and employer components. In two later articles, Abowd-Kramarz[2000a,b] apply new methods that permit to use the exact solution to the estimation problem. Abowd-Kramarz[2000a] analyze the same American data as Abowd-Finer-Kramarz[1999] whereas in a companion paper, Abowd-Kramarz[2000b] examine the same French data as Abowd-Kramarz-Margolis[1999]. Generally speaking, the authors find in the two later studies some results consistent with the two early studies, so the following presentation will concentrated on Abowd-Kramarz[2000a,b]. See appendix A for a presentation of their statistical model.

Using data from the State of Washington(CWBH 1984-1993), Abowd-Kramarz[2000a,b] show that person effects (net of observable non-time-varying characteristics) explain about half of the raw inter-industry wage differential and about 30 percent of the firm-size wage differential. Firm heterogeneity accounts for half of the raw inter-industry wage differential and about 70 percent of the firm-size wage differential. The results for the State of Washington are compared to those for the France. Using French linked employer-employee data(DADS 1976-1980,1982,1984-1987), Abowd-Kramarz[2000a,b] find that inter-industry wage differentials are due almost in equal proportions to unmeasured individual and employer heterogeneity while firm-size wage differentials are due primarily to unmeasured firm heterogeneity. These results are summarized in table 1.1. Further, Abowd-Kramarz[2000a,b] verify that a regression of worker and firm effects on the raw industry wage differentials across industries generates an \( R^2 \) in excess of 0.98 for the State of Washington and 0.96 for the France. Finally, the two unobservable components are not highly correlated: the correlation coefficient is -0.005 for the State of Washington and 0.08 for the France.

The implications of the path-breaking work of Abowd-Kramarz[2000a,b] are multiple. First, observed wage differentials reflect, to a great extent, dif-
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Table 1.1: Decomposition of wage differentials in Abowd-Kramarz[2000a,b]

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<th>Country</th>
<th>Raw inter-industry wage differentials</th>
<th>Raw firm-size wage differentials</th>
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<tr>
<td>France</td>
<td>Average firm effect 45%</td>
<td>75%</td>
</tr>
<tr>
<td></td>
<td>Average person effect 55%</td>
<td>25%</td>
</tr>
<tr>
<td>U.S.</td>
<td>Average firm effect 50%</td>
<td>70%</td>
</tr>
<tr>
<td></td>
<td>Average person effect 50%</td>
<td>30%</td>
</tr>
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</table>

Table 1.2: Decomposition of log-wage variance in Postel-Vinay and Robin[2002]

<table>
<thead>
<tr>
<th>Occupation</th>
<th>Firm effect</th>
<th>Friction effect</th>
<th>Person effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Executives, managers and engineers</td>
<td>19.4%</td>
<td>38.7%</td>
<td>41.9%</td>
</tr>
<tr>
<td>Supervisors, administrative and sales</td>
<td>27.9%</td>
<td>55.1%</td>
<td>17.8%</td>
</tr>
<tr>
<td>Technical supervisors and technicians</td>
<td>32.8%</td>
<td>60.6%</td>
<td>6.6%</td>
</tr>
<tr>
<td>Administrative support</td>
<td>34.6%</td>
<td>55.7%</td>
<td>9.7%</td>
</tr>
<tr>
<td>Skilled manual workers</td>
<td>41.5%</td>
<td>58.5%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Sales and service workers</td>
<td>37.1%</td>
<td>57.9%</td>
<td>5.0%</td>
</tr>
<tr>
<td>Unskilled manual workers</td>
<td>40.8%</td>
<td>59.2%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>
ferences in observable firm characteristics such as sector of activity, firm size, productivity, etc. Second, their studies provide evidence for the importance of both unobservable firm and person differences as complementary sources of wage differentials, and invite to assess the origin of the unobservable person or firm effects. The work of Postel-Vinay and Robin[2002], Cahuc, Postel-Vinay and Robin[2006], Mortensen[2002a], Gibbons-Waldman[1999a,b,2003] implies that person effects are generally associated with unobserved, possibly firm- or match-specific, worker quality whereas firm effects are generally associated with firm-specific compensation policies. Third, the work of Abowd-Kramarz[2000a,b] strengthens Oi-Idson’s[1999] claim that most of the firm-size wage differential is a firm effect. While personal heterogeneity clearly plays a role (30% of the differential is significant), the search for explanations of this phenomenon should clearly favor models that relate the firm-size to the firm’s structural compensation decisions.

The work of Abowd-Kramarz[2000a,b] is appealing. However, Postel-Vinay and Robin[2002] follow another way to investigate the underlying determinants of the wage variance. The authors construct and structurally estimate an equilibrium search model with on-the-job-search. Workers differ in ability and firms differ in the marginal productivity of efficient labor. Firms make take-it-or-leave-it wage offers to workers conditional on their characteristics. All employers pay workers hired from unemployment their reservation wage, but compete with any alternative employer that the worker meets subsequently. When an employed worker has an outside employment opportunity, the employer offering the alternative job and the worker’s current employer bid against each other until the expected future value of either hiring or keeping the worker is zero for one of the two. Their model then delivers a theory of steady-state wage dispersion driven by heterogenous worker abilities and firm productivity, as well as by matching frictions(see section 1.6.1 for detailed presentation of the model).

Postel-Vinay and Robin[2002] then estimate the model using matched employer and employee French panel data(DADS 1996-1998), and provide a
decomposition of cross-employee wage variance:

\[
V(\ln(w)) = V_p[E(\ln(w)|p)] + E_p[V(\ln(w)|p)] \\
= V_p[E(\ln(w)|p)] + V(\ln(\varepsilon)) + (E_p[V(\ln(w)|p]) - V(\ln(\varepsilon)))
\]

where \( w \) represents the wage, \( p \) the productivity, \( \varepsilon \) the ability of the worker. In Postel-Vinay and Robin[2002], the first term, \( V_p[E(\ln(w)|p)] \), is the between-firm wage variance. It reflects the fact that some firms pay higher wages on average and thus contribute to individual wage dispersion. Hence, it is quite natural to label this term the “firm effect”. The second term, \( V(\ln(\varepsilon)) \), is obviously interpreted as the contribution of dispersion in unobserved individual ability. The authors therefore refer to it as the “person effect”. The third term, \( (E_p[V(\ln(w)|p]) - V(\ln(\varepsilon))) \), is the share of the within-firm wage variance which is not due to dispersion in individual ability. Its origin is identified in the model: the reason why two identical workers working at identical firms can earn different wages is that the two workers had different draws of alternative wage offers. The authors then interpret this term as the “effect of market frictions”.

Postel-Vinay and Robin[2002] obtain a remarkable result: individual ability differences explain about 40% of the log wage variance for executives, managers and engineers, 20% for workers with lower executive functions, 10% for technicians and technical supervisors and the administrative support staff, and virtually nothing for the other categories. It therefore seems that the more skill-intensive an occupation is, the more heterogeneous is the category of workers who can apply to it. At the bottom of the skill hierarchy, manual workers and employees are rather homogeneous. Further, the contribution of market imperfections to wage dispersion is typically around 60%. These results are reported in table 1.2.

Nevertheless, the estimates of the person effect in Postel-Vinay and Robin[2002] contrast a lot with those in Abowd-Kramarz[2000a,b] who use the same data but over a different period of time. Why so? Postel-Vinay and Robin[2002] answer: because of labor market frictions. Endogenous worker mobility through the sequential sampling of alternative job offers creates earnings differentials across identical workers working at identical firms. The authors
1.2. EMPIRICAL EVIDENCES

find that the explanatory power of historical differences is far from negligible as they account for 60% of total wage variance. The contribution of market imperfections to wage dispersion is isolated by Postel-Vinay and Robin[2002] as the share of the within-firm wage variance which is not due to dispersion in individual ability, hence named the friction effect. On the opposite, it is regarded in Abowd-Kramarz[2000a,b] as part of the share of within-industry wage differentials that are not explained by differences in firms characteristics, and treated as part of the person effect. Therefore, Postel-Vinay and Robin[2002] state that Abowd-Kramarz[2000a,b] over-estimate the contribution of the inobservable person effect to wage differentials.

This argument is pertinent. Search friction is the first determinant of wage dispersion. In a hypothetical frictionless market, all workers will instantaneously find the highest wage offer and all firms will instantaneously fill a vacancy, there will be neither unemployment nor vacant jobs in economics. Nevertheless, if Abowd-Kramarz[2000a,b] really neglect the contribution of market friction to wage differentials, the estimates of the friction effect in Postel-Vinay and Robin[2002] are, perhaps, also discussible.

Mortensen[2002a] questions why firms have complete information in Postel-Vinay and Robin[2002]. However, a purely empirical query about the results of decomposition of log-wage variance also exists. After all, a 60% share of friction effect is huge, which means it is market frictions, but not firm heterogeneity (measured and unmeasured), that is the most important determinant of wage differentials. Further, the estimate of Postel-Vinay and Robin[2002], a 60% share of friction effect, leads to query why most of equilibrium search models (including Burdett-Mortensen[1998] and Postel-Vinay and Robin[2002]) could not well explain wage dispersion in an environment where all firms and workers are respectively identical.

The comments above invite to assess the quantitative importance of frictions. Interestingly, it is Bontemps-Robin-Van den Berg[2000] who first answers this question. Using French data (French Labor Survey 1990-1993), the authors estimate the Burdett-Mortensen[1998] model with continuous productivity dispersion, and find that the model provides a close fit to the wage dispersion.

---

6See sections 1.3 and 1.6.1 for presentations of these two models.
data. In contrast, the model without productivity dispersion, i.e., the model in which wage dispersion is entirely due to market frictions and wage policy of the firm, cannot explain more than 10% of total wage variation in the data. This result clearly indicates that Bontemps-Robin-Van den Berg[2000] have already obtained, in the framework of equilibrium search theory, an exact evaluation of the upper limit the pure friction effect to wage variation.

Another investigation is found in Van den Berg-Van Vuuren[2003] who designedly study the effect of search frictions on (mean) wage. Using Danish data(IDA 1980-1994), the authors estimate a mean wage equation of the form:

\[ E_i(w_m) = \alpha_0 + \alpha_1 E_i(p_m) + \alpha_2 \log(k_m + 1) + \varepsilon_m \]

where the endogenous variable of interest is the mean wage across firms in a market. Let indices \( m \) and \( i \) denote the market \( m \) and the firm \( i \). The endogenous variable is then denoted by \( E_i(w_{mi}) \) and the explanatory variables are the mean productivity across firms \( E_i(p_{mi}) \) and the measure of friction \( \log(k_m + 1) \).

The basic measure of friction is \( k = \lambda_1/\delta \) where \( \lambda_1 \) is the job contact rate when employed and \( \delta \) denotes the job destruction rate. This measure is an indicator of the relative power of workers vis-a-vis employers. Indeed, the authors also use some alternative measures such as \( \log(k) \), \( \log(\lambda_1) \) or \( \log(\delta) \) in the mean wage equation. Three transition parameters, \( \lambda_1, \lambda_0 \) and \( \delta \) are estimated in the basic partial job search framework provided that the distributions of wage offers and earnings, \( F(w) \) and \( G(w) \), are well identified from the data. The parameter \( \lambda_0 \) denotes the job contact rate when unemployed and is interpreted as the exit rate out of unemployment. It should be particularly stressed that the authors do not impose a specific full equilibrium search model (like Bontemps-Robin-Van den Berg[2000]) on the data, and do not make any assumption on the effect of search frictions on wages.

The interpretation of this mean wage equation is as follows. The authors analyze the wage determination in a benchmark equilibrium job search model(Burdett-Mortensen[1998] with continuous productivity dispersion) and a prototype equilibrium search-matching model(Pissarides[1990]). They find that the mean wage of either offers \( E(w|w \sim F(w)) \) or earnings \( E(w|w \sim G(w)) \).
is affected by three additive factors: mean productivity and mandatory minimum wage, market frictions, and an interaction effect between market frictions and productivity dispersion among firms. Because Denmark has no clearly defined or observable minimum wage, the authors ignore institutional wage floors as an explanatory variable of the mean wage across firms. To assess the impact of market frictions to wages, the authors therefore construct the above mean wage equation and test whether the sign of $\alpha_2$ is positive.

The empirical results are unambiguous. Frictions have a significant effect on the mean wage in the market: higher frictions imply that the mean wage across firms is lower. Further, Van den Berg-Van Vuuren[2003] assess the quantitative importance of frictions by examining the fraction of wage variation that can be explained by frictions. They first decompose the total wage variation across firms into variation within markets and variation between markets. They find the former explains 62%, so sector and region explain 38% of wage variation across firms. Next, they decompose the total “between-market” variation of the market-specific mean wage into variation due to differences in frictions across markets, variation due to differences in the market-specific mean productivity, and residual variation. These decomposition results invariably state that less than 5% of the between-market variation is due to differences in frictions, while at most another 5% can be attributed to interactions between frictions and the mean productivity. The authors then conclude that inter-industry (and inter-region and inter-skill) wage differences cannot be explained by differences in the degree of frictions.

In sum, Van den Berg-Van Vuuren[2003] using Danish data and Bontemps-Robin-Van den Berg[2000] using French data all indicate the pure friction effect typically explains no more than 10% of wage variation in the data. Productivity variation across firms within a market could by itself not generate any wage dispersion, in the sense that wage dispersion may equal zero if frictions are infinitely large or absent. It is rather the interaction between productivity variation and frictions that provides a good fit to within-market wage distributions. Consequently, we could be sure to say that Postel-Vinay and Robin[2002] have over-estimated the pure friction effect. The 60% share of “friction effect” in Postel-Vinay and Robin[2002]—the within-firm wage
variance which is not due to dispersion in individual ability—is rather attributed to an interaction effect between market friction and firm/worker heterogeneity.

1.3 Pure wage dispersion

From this section, we turn to survey the literature that aims at giving theoretical explanations of wage dispersion. Most theories explain wage dispersion along four lines: pure wage dispersion, wage dispersion linked with exogenous firm/person heterogeneity, wage dispersion linked with endogenous firm/person heterogeneity, intra-firm wage dispersion and wage growth. The theory of pure wage dispersion will be first presented in this section.

1.3.1 Introduction

The basic ideas of the basic Burdett-Mortensen[1998] model are as follows. All firms and all workers are respectively identical. Search is random, sequential and non-directed. Unemployed workers search for a job and, more importantly, employees search for a better job. Firms make take-it-or-leave-it wage offers to workers. The wage posting approach is consistent with the idea that each employer is free to choose a particular wage policy, say to be either a “high-” or a “low-” wage firm. Given the wages offered by all others and the distribution of worker reservation wage rates, the labor force available to a specific employer evolves in response to the employer’s wage. The higher the wage the larger the steady-state labor force, because higher wage firms are more attractive to outsiders and retain insiders more readily. The resulting labor supply relation determines the profit of each employer conditional on the wage offered by other employers and the reservation wages demanded by workers. Facing the same trade off between wage and firm size, some firms choose the high wage even though profit generated per worker is lower, making up the difference in high volume. Consequently, wage dispersion exists in equilibrium even when workers are equally productive in all jobs. Wage dispersion of this type is called “pure” wage dispersion: any form of heterogeneity is excluded.
1.3. PURE WAGE DISPERSION

1.3.2 Model

1.3.2.1 Labor market flows

Suppose a labor market in steady state where all workers and firms are respectively identical ex ante. At a moment in time, a worker is either unemployed (state 0) or employed (state 1). Unemployed workers search for a job and employees search for a better job. Allowing the arrival rate of job offer to depend on a worker’s current state, let \( \lambda_i, i = 0, 1 \), representing the parameters of a Poisson arrival processus, denote the job contact rate while a worker is currently occupying state \( i \).

Let normalize to one the sum of unemployed workers and employed workers. Let \( u \) denote the steady-state number of unemployed workers. In the steady state, the flow of workers into employment, \( \lambda_0u \), equals the flow from employment to unemployment, \( \delta(1 - u) \) where \( \delta \) represents the destruction rate of a job-worker match. Given this fact, the steady state unemployment rate is deduced from:

\[
(1 - u)\delta = \lambda_0u
\]

Let \( G \) and \( F \) denote the cumulative distribution functions (c.d.f.) of wage earnings and offers, respectively. The steady-state number of employees earning \( w \) or less is \( (1 - u)G(w) \). Employees leave this stock either because they are laid off (which happens at rate \( \delta \)), or because they receive an outside offer with associated wage greater than \( w \) (at rate \( \lambda_1[1 - F(w)] \)). On the other side, unemployed workers contact a offer \( w \) or less at rate \( \lambda_0F(w) \). The steady-state distribution of wage earnings, \( G(w) \), is then derived from the following equilibrium flows condition:

\[
u\lambda_0F(w) = (1 - u)G(w)[\delta + \lambda_1(1 - F(w))]\]

Let define \( k_0 = \frac{\lambda_0}{\delta}, k_1 = \frac{\lambda_1}{\delta} \). These two indicators represent the ratios of state-dependent job contact rate to the job separation rate. The unique steady-state distribution of wages earned by employed workers can be written as:

\[
G(w) = \frac{F(w)}{1 + k_1(1 - F(w))}
\]

(1.1)
1.3.2.2 Worker search behavior and reservation wage determination

Let $U$ and $W$ represent the expected discounted lifetime income of unemployed and employed workers, respectively. The opportunity cost of searching while unemployment, the interest rate on its asset value, is equal to income while unemployment, $b$, plus the expected capital gain attributable to searching for an acceptable job where acceptance occurs only if the value of employment, $W$, exceeds that of continued search. The reservation wage, denoted by $R$, is the wage rate that makes an unemployed worker indifferent between staying in unemployment and working as employee. In equilibrium, no employer will offer a wage less than $R$ because any employer offering such wage would have no employees.

At a moment in time, an employee currently employed at wage $w$ searches on-the-job and expects a future wage increase, but envisages the firing risk. So formally, the value functions of the workers solve the following asset pricing equations:

$$rU = b + \lambda_0 \int_{U}^{W} [W(y) - U] dF(y)$$

$$rW(w) = w + \delta [U - W(w)] + \lambda_1 \int_{w}^{W} [W(y) - W(w)] dF(y)$$

The reservation wage, defined by $W(R) = U$, can be written as:

$$R = b + (\lambda_0 - \lambda_1) \int_{U}^{W} [W(y) - U] dF(y)$$

$$= b + (\lambda_0 - \lambda_1) \int_{R}^{\infty} \left[ \frac{1 - F(y)}{r + \delta + \lambda_1(1 - F(y))} \right] dy$$

by an integration by parts. Keeping the analysis as simple as possible, we consider the case where $r \to 0$. So:

$$R = b + (k_0 - k_1) \int_{R}^{\infty} \left[ \frac{1 - F(y)}{1 + k_1(1 - F(y))} \right] dyF(y) \quad (1.2)$$

1.3.2.3 Wage and recruitment policies of the firm

Firm behavior is now considered. There is a large number of firms in economics, each firm consists of many jobs. The steady-state number of workers employed at wage interval $[w - \varepsilon, w]$ where $\varepsilon > 0$, is represented by
1.3. PURE WAGE DISPERSION

\[ G(w) - G(w - \varepsilon)(1 - u), \text{ whereas } [F(w) - F(w - \varepsilon)] \text{ is the measure of firms offering wages in this interval. Thus, the measure of workers per firm earning a wage } w \text{ can be expressed as:} \]

\[ l(w) = \lim_{\varepsilon \to 0} \frac{G(w) - G(w - \varepsilon)}{F(w) - F(w - \varepsilon)} (1 - u) \]

\[ = \frac{G'(w)}{F'(w)} (1 - u) \]

\[ = \frac{k_0(1 + k_1)}{(1 + k_0)(1 + k_1 - F(w))^2} \]

Let \( p \) denote the flow of revenue generated per employed worker. Hence, an employer’s steady-state profit given the wage offer \( w \) can be written as:

\[ \pi = \max_w (p - w)l(w) \]

Because all firms are identical, every optimal wage offer must yield the same steady-state profit. The iso-profit condition, \( \pi = (p - R)l(R) = (p - w)l(w) \), requires:

\[ F(w) = \left( \frac{1 + k_1}{k_1} \right) \left[ 1 - \left( \frac{p - w}{p - R} \right)^{1/2} \right] \]  
\[ (1.3) \]

The highest wage paid, denoted by \( \bar{w} \), is deduced from the boundary condition \( F(\bar{w}) = 1 \),

\[ \bar{w} = p - (p - R)/(1 + k_1)^2 \]  
\[ (1.4) \]

Substituting (1.3) into equation (1.2) gives:

\[ R = b + \left( \frac{k_0 - k_1}{k_1} \right) \int_R^{\bar{w}} \left[ 1 - \left( \frac{1}{1 + k_1} \right) \left( \frac{p - y}{p - R} \right)^{-1/2} \right] dy \]

\[ = b + \left( \frac{k_0 - k_1}{k_1} \right) \left[ \bar{w} - R + \frac{2(p - R)}{1 + k_1} \left( \left( \frac{p - \bar{w}}{p - R} \right)^{1/2} - 1 \right) \right] \]

Substituting once again (1.4) into the above equation yields:

\[ R = \frac{(1 + k_1)^2b + (k_0 - k_1)k_1p}{(1 + k_1)^2 + (k_0 - k_1)k_1} \]  
\[ (1.5) \]
1.3.2.4 Discussion

Up to this point, a non-degenerated wage dispersion is already well defined in equilibrium. The equilibrium wage offer distribution derived here contrasts sharply with both the competitive Bertrand solution and the Diamond’s[1971] pure strategy solution to the wage-posting game. Still, both are limiting cases. Specially, as $k_1 \to 0$, equations (1.4) and (1.5) imply that both the highest wage and the lowest wage in the market converge to the opportunity cost of unemployment $b$. Hence, Diamond’s solution is obtained in this limit. Allowing the possibility that employed workers search on-the-job and move from lower to higher paying jobs resolves the Diamond paradox. In another limiting case, as friction vanish in the sense that $k_1 \to \infty$, equation (1.1) requires that all wage offers must converge to a mass point at the highest wage, whereas equation (1.4) implies that the highest wage limits to the productivity $p$. Hence, the competitive equilibrium constitutes another limiting solution of the Burdett-Mortensen[1998] model.

Therefore, wage dispersion exists in equilibrium even when workers are equally productive in all jobs. Figure 1.2 plots the equilibrium distributions of wage offers and earnings, predicted by the model. The parameters of the models are calibrated to reflect French economy. Three further strong predictions also follow from this simplest version of the model. First, more
1.4. EXOGENOUS HETEROGENEITY

experienced workers and those with more tenure are more likely to be found in higher paying jobs. Second, there is a positive association between the labor force size and the wage paid. Finally, there is also a negative relationship between wage offers and quit rates across employers.

It should be pointed out that this model has problems fitting empirical wage data. In particular, the model implies that the equilibrium offer and earnings densities increase monotonously across their common support in apparent conflict with the single modal shape typically observed. So in order to match the observed wage dispersion, we need to introduce some form of heterogeneity into the model.

1.4 Wage dispersion linked with exogenous heterogeneity

1.4.1 Heterogeneity in unemployment compensation

1.4.1.1 Introduction

One implication of the Diamond’s [1971] paradox is that market failure is the inevitable consequence of costly search and market power on one side of the market. Albrecht-Axell [1984] show that this conclusion is a consequence of the assumption that all workers have identical search and opportunity costs of employment. Instead, if workers have different unemployment benefits or search costs, employers will pay different types of workers different wages (their reservation wages), so that wage dispersion exists in equilibrium even though the support of the distribution is discrete. Mortensen [1990] integrates Albrecht-Axell’s [1984] assumption of different opportunity costs of employment into the equilibrium search framework. In Mortensen [1990], wages other than those on the support of reservation wage distribution are offered, but still the wage offer support generally is not continuous. Burdett-Mortensen [1998] first derive closed form offer distribution in the case of continuous distribution of unemployment compensation. The essence of their model follows.
1.4.1.2 Model

**Labor market flows** Assume all job-worker matches are equally productive. Workers, however, differ in how they value unemployment. In particular, let $H(b)$ denote the proportion of workers whose opportunity cost of employment is no greater than $b$. Assume $H(b)$ is continuous and defined on $b \in [b, \bar{b}]$. To simplify the analysis, assume that the job arrival rate is independent of employment status, that is, $\lambda_0 = \lambda_1 = \lambda$, so that the reservation wage of a type $b$ worker is $R = b$. In this case, the steady-state measure of unemployed workers willing to accept a wage offer less than or equal to $b$ is:

$$u(b) = \int_b^b \left( \frac{\delta}{\delta + \lambda[1 - F(x)]} \right) dH(x)$$

since the unemployment rate of worker of type $x$ is $\frac{\delta}{\delta + \lambda[1 - F(x)]}$. Equivalently, the above equation implies:

$$H(b) = \int_b^b \left( \frac{\delta + \lambda[1 - F(x)]}{\delta} \right) du(x) \quad (1.6)$$

Let the steady-state number of workers employed by employers offering a wage no greater than $w$ be given by $(1 - u)G(w)$, where $u \equiv u(b)$ is total unemployment. In a steady-state the flow of workers leaving employers offering a wage no greater than $w$ equals the flow of workers entering such employers,

$$(\delta + \lambda[1 - F(w)])(1 - u)G(w) = \lambda \int_b^w [F(w) - F(x)] du(x)$$

where $\delta + \lambda[1 - F(w)]$ is the match separation rate, and $F(w) - F(x)$ is the probability that an offer received by a type $x$ worker is acceptable whose opportunity cost of employment is no greater than $w$.

Again let’s define $k = \lambda/\delta$. Differentiating the above equation with respect to $w$ yields:

$$(1 - u)G'(w) = \frac{kF'(w) \left\{ u'(w)[1 + k[1 - F(w)]] + k \int_b^w [F(w) - F(x)] du(x) \right\}}{(1 + k[1 - F(w)])^2}$$

$$= \frac{kF'(w) \int_b^w [1 + k(1 - F(x))] du(x)}{(1 + k[1 - F(w)])^2}$$

$$= \frac{kF'(w)H(w)}{(1 + k[1 - F(w)])^2} \quad (1.7)$$
### 1.4. EXOGENOUS HETEROGENEITY

by equation (1.6).

**Wage and recruitment policies of the firm** The measure of workers per firm earning a wage $w$ is:

$$l(w) = \frac{G'(w)}{F'(w)}(1 - u)
= \frac{kH(w)}{(1 + k[1 - F(w)])^2}$$

Let $p$ denote the flow of revenue generated per employed worker. Hence, an employer’s steady-state profit given the wage offer $w$ can be written as:

$$\pi = \max_w (p - w)l(w)$$

The iso-profit condition, $\pi = (p - w)l(w) = (p - w)l(w)$, requires:

$$(p - w)\frac{H(w)}{(1 + k)^2} = (p - w)\frac{H(w)}{(1 + k[1 - F(w)])^2}$$

that is

$$F(w) = \left(\frac{1 + k}{k}\right)\left(1 - \sqrt{\frac{(p - w)H(w)}{(p - w)H(w)}}\right)$$

In market equilibrium, the optimal lowest wage offer, denoted by $w$, satisfies:

$$w = \arg \max \{\pi(w) = (p - w)l(w)\}$$

The first order condition of optimality requires:

$$\frac{H'(w)}{H(w)} = \frac{1}{p - w}$$

To give a quantitative illustration of this model, assume that $H(b)$ follows a Pareto distribution function:

$$H(b) = 1 - \frac{b}{b}^\beta$$

where $b \in [\underline{b}, \bar{b}]$. In this case, the lower bound of wage support is the unique solution of

$$p - w = \frac{H(w)}{H'(w)} = \frac{1 - (b/w)^\beta}{\beta b^\beta w^{-\beta - 1}}$$

(1.9)
Figure 1.3: Equilibrium wage distributions (heterogenous workers)

Illustrative calibration: $\lambda = 0.3$, $\delta = 0.1$, $p = 5.5$, $b = 1$, $\beta = 4$

The highest wage offer, $\bar{w}$, is deduced from the boundary condition $F(\bar{w}) = 1$. Formally, $\bar{w}$ soles:

$$\frac{(p - \bar{w})H(\bar{w})}{(p - \bar{w})H(\bar{w})} = \left(\frac{1}{1 + k}\right)^2$$  \hspace{1cm} (1.10)

1.4.1.3 Discussion

Figure 1.3 plots the equilibrium distributions of wage offers and earnings, predicted by the model. Note that equation (1.8) reduces to (1.3) if we assume that workers have the same opportunity costs of employment. Therefore, equilibrium wage distribution in this model still has an increasing density with a left skew not so evident as in the case of homogenous workers and firms.

This model implies that heterogeneity in observable worker characteristics (such as unemployment compensation) is, by itself, not sufficient to generate offer and wage densities with the shape properties approximate to those actually observed.
1.4. HETEROGENEITY IN FIRM PRODUCTIVITY

1.4.2.1 Introduction

As emphasized by Abowd-Kramarz[2000a,b], observed wage differentials reflect, to a great extent, differences in employer productivity. Bontemps-Robin-Van den Berg[2000] establish that the Burdett-Mortensen[1998] model could allow for a closer fit to the actual wage data once differences in employer productivity are considered. In what follows, the basic Burdett-Mortensen[1998] model is generalized to allow for a continuous distribution of productivity types (see Burdett-Mortensen[1998], Bontemps-Robin-Van den Berg[2000]).

1.4.2.2 Model

Labor market flows Suppose a labor market in steady state where all workers are identical ex ante. Unemployed workers search for jobs whereas employees search on-the-job for better outside offers. To simplify the analysis, assume that the job arrival rate is independent of employment status, that is, \( \lambda_0 = \lambda_1 = \lambda \), so that the reservation of the worker is given by \( R = b \).

Worker search behavior and labor turnover flows in this model are the same as described in the basic Burdett-Mortensen[1998] model. We simply restate here the equilibrium conditions:

\[
\begin{align*}
    u &= \frac{\delta}{\delta + \lambda} \\
    G(w) &= \frac{F(w)}{1 + k(1 - F(w))}
\end{align*}
\]

Note that \( k = \lambda/\delta \).

Wage and recruitment policies of the firm The measure of workers per firm earning a wage \( w \) is:

\[
\begin{align*}
    l(w) &= \frac{G'(w)}{F'(w)}(1 - u) \\
    &= \frac{kH(w)}{(1 + k[1 - F(w)])^2}
\end{align*}
\]
Firms differ in the technologies that they operate. Assume that firms differ by an exogenous technological parameter $p$ which follows a distribution $\Gamma$ across firms over the support $p \in [p_{\text{min}}, p_{\text{max}}]$. This distribution is assumed continuous with density $\gamma$. In this case there is a unique wage associated with each productivity type. This outcome derives from the fact that the market distribution of wage offers is a transformation of the underlying distribution of productivity types. In other words, the fraction of offers equal or less than $w(p)$ is simply the proportion of firms with productivity $p$ or less. Formally,

$$F(w(p)) = \Gamma(p) \quad (1.11)$$

Differentiating this equation with respect to $w$ gives:

$$w'(p) = \frac{\Gamma'(p)}{F'(w(p))} > 0$$

This equation means that more productive employers offer higher wages. For any employer of productivity type $p > p$, the steady-state profit is determined by a wage choice:

$$\pi = \max_w (p - w)l(w)$$

The first-order condition for an interior solution is:

$$\frac{1}{p - w} = \frac{l'(w)}{l(w)} = \frac{2kF'(w)}{1 + k(1 - F(w))}$$

Easy to verify that the second-order condition is satisfied. After substituting $F(w(p))$ and $F'(w(p))$ out of the above equation (by use of (1.11)) and rearranging the terms, one obtains:

$$w'(p) = \Gamma'(p) \frac{2k[p - w(p)]}{1 + k_1(1 - \Gamma(p))} \quad (1.12)$$

Also assume that equilibrium wage offers are bounded by a mandatory minimum wage which satisfies $w_{\text{min}} \geq b$ where $b$ equals the reservation wage.

A primary characteristic of this model is that more productive employers offer higher wages ($w'(p) > 0$), make more employees ($l'(p) > 0$), and retrieve higher profit ($\pi'(p) > 0$). Indeed, assume that $p_{\text{min}} \leq p_1 < p_2 \leq p_{\text{max}}$. It follows that:

$$\pi_2(p_2) = (p_2 - w_2)l(w_2) \geq (p_2 - w_1)l(w_1)$$

$$> (p_1 - w_1)l(w_1) = \pi_1(p_1)$$
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Figure 1.4: Equilibrium wage distributions (productivity dispersion)

Illustrative calibration: $\lambda_0 = 0.7$, $\lambda_1 = 0.3$, $\delta = 0.1$, $w = 1$, $b = 0.8$, $p = 2$, $\beta = 4$

where the first inequality is implied by the optimality of wage offers.

1.4.2.3 Discussion

In line with Mortensen[2002a], we suppose that $\Gamma(p)$ follows a Pareto distribution:

$$\Gamma(p) = 1 - \left(\frac{p_{\text{min}}}{p}\right)^\beta$$

Figure 1.4 plots the equilibrium distributions of wage offers and earnings, predicted by the model.

Clearly, this model with continuous productivity dispersion permits to generate a uni-modal wage density. In fact, as

$$f(w(p)) = \frac{1 + k_1(1 - \Gamma(p))}{2k_1(p - w(p))}$$
$$g(w(p)) = \frac{(1 + k_1)f(w(p))}{(1 + k_1(1 - \Gamma(p)))^2}$$

and $w'(p_{\text{max}}) = 0$ if $\Gamma'(p_{\text{max}}) = 0$ from equation (1.12), any continuous productivity distribution with a vanishing right tail does generate an offer distribution with a declining density for all large enough wage rates($f(w(p_{\text{max}})) \to 0$). Still, a highly skewed distribution of productivity is generally required to
generate offer and wage densities with the shape properties approximate to those actually observed.

Bontemps-Robin-Van den Berg[2000] using French data(FLS 1990-1993) structurally estimate this model and find that the model gives a close fit to the wage data. In contrast, the model without productivity dispersion cannot explain more than 10% of total wage variation in the data.

1.5 Wage dispersion linked with endogenous heterogeneity

1.5.1 Endogenous search effort

1.5.1.1 Introduction

Up to this point, workers receive offers at rates that are independent of their own decisions. The only worker decision of interest is to accept or not wage offers as they arrive. Since the worker’s incentive to seek a higher paying job is closely linked with the perspective of wage increase, the assumption that workers contact job at exogenous rates is inappropriate. Mortensen[2002b] generalizes the basic Burdett-Mortensen[1998] model to incorporate a worker search effort from this viewpoint(see also Christiansen et al.[2005]). The essence of the model follows.

1.5.1.2 Model

**Labor market flows** All workers and firms are respectively identical. Suppose that offers arrive at a Poisson frequency which is proportional to search effort. Specifically, let $\lambda s$ represent the offer arrival rate when employed where $s$ is search effort and $\lambda$ is a contact parameter, $\lambda s_0$ represent the offer arrival rate when unemployed where $s_0$ is search effort of the unemployed. In the steady state, the balance between flows into and out of unemployment is written as:

$$(1 - u)\delta = u\lambda s_0$$

where $\delta$ is the job destruction rate.
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Let $G$ and $F$ denote the distributions of wage earnings and offers, respectively. The stock of the workers employed at wage $w$ or less is $(1 - u)G(w)$. At a moment in time, workers leave this stock because they are laid off (at rate $\delta$), or because they receive an outside wage offer greater than $w$ (at rate $\lambda [1 - F(w)] \int_w^\infty s(x)dG(x)$). On the other side, the inflows into the stock $(1 - u)G(w)$ consist of the unemployed who draw no greater than $w$ (at rate $\lambda s_0 F(w)$). In steady-state, the balance between flows into and out of this stock is written as:

$$(1 - u) \left[ \delta G(w) + \lambda [1 - F(w)] \int_w^w s(x)dG(x) \right] = u \lambda s_0 F(w)$$

Equivalently,

$$\frac{F(w) - G(w)}{1 - F(w)} = \frac{\lambda}{\delta} \int_w^w s(x)dG(x) \quad (1.13)$$

The horizontal difference between the distribution of wages earned, $G(w)$, and the distribution of wage offers, $F(w)$, represents an employment effect. Note that $F(w) - G(w) = [1 - G(w)] - [1 - F(w)]$. The difference is positive because an employed worker is more likely to find a higher paying job during her current employment spell while a worker hired from unemployment earns a random draw from the wage offer distribution. Equation (1.13) implies that the horizontal differences between the wage offer and wage earned distribution functions depend on the extent of friction in the market as reflected in the ratio of the offer arrival rate to the job separation rate and the average search effort of the workers earning no more than $w$.

Search behavior of the worker Again let $U$ and $W$ denote the expected discounted lifetime income of unemployed and employed workers, respectively. All workers search for job offers, however search is costly. Let $C(\cdot)$ denote the cost of search effort, which is assumed to be an increasing convex function. Unemployed workers receive unemployment compensation $b$ and search for job offers. Similarly, employed workers receive the wage, search for a better outside offer but envisage the firing risk. Formally, the value of
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Employment and unemployment are respectively written as:

\[ rU = \max_{s \geq 0} \left\{ b - C(s_0) + \lambda s_0 \int_R^w [W(x) - U] dF(x) \right\} \]

\[ rW(w) = \max_{s \geq 0} \left\{ w - C(s) + \delta [U - W(w)] + \lambda s \int_w^w [W(x) - W(w)] dF(x) \right\} \]

Clearly, an employee’s expected lifetime income increases with his current wage rate, as

\[ W'(w) = \frac{1}{r + \delta + \lambda s(w)[1 - F(w)]} > 0 \]

The reservation wage of the unemployed, \( R \), equates the value of unemployment to the value of employment, i.e., \( W(R) = U \). As a consequence, the above two value functions together imply that the reservation wage is equal to the unemployment benefit,

\[ R = b \]

and that the search intensity of an unemployed worker is the same as that of a worker employed at the reservation wage provided that the reservation wage is the unemployment benefit,

\[ s_0 = s(R) \]

The optimal search effort of the employee must satisfy the first order condition:

\[ c'(s) = \lambda \int_w^w [W(x) - W(w)] dF(x) \]

\[ = \lambda \int_w^w W'(x) [1 - F(x)] dx \]

\[ = \int_w^w \frac{\lambda [1 - F(x)]}{r + \delta + \lambda s(x)[1 - F(x)]} dx \]

so \( s = s(w) \). Differentiating the optimal condition with respect to \( w \) gives:

\[ c''(s(w))s'(w) = \frac{-\lambda [1 - F(w)]}{r + \delta + \lambda s(w)[1 - F(w)]} \]

(1.14)

Because the likelihood of finding a better job declines with the wage earned, search intensity declines with an employed worker’s current wage, i.e., \( s'(w) < 0 \). Also because the highest-paid workers have no incentive to search on-the-job, their search intensity is zero, that is, \( s(w) = 0 \).
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Wage and recruitment policies of the firm We first consider the recruitment policy. Because any unemployed worker will accept a wage no less than $R$ but an employed worker accepts only if currently paid less than the wage offered, the unconditional probability that a randomly contacted worker will accept is:

$$h(w) = \frac{u\lambda s_0 + (1-u)\lambda \int_0^w s(x)dG(x)}{u\lambda s_0 + (1-u)\lambda \int_0^w s(x)dG(x)}$$

Differentiating this equation with respect to $w$ gives

$$h'(w) = \frac{\lambda s(w)G'(w)}{\delta + \lambda \int_0^w s(x)dG(x)} = \frac{\lambda s(w)F'(w)}{\delta + \lambda s(w)[1 - F(w)]}$$

Let $p$ represent the employer’s productivity, and $J(w)$ denote the expected present value of the future incoming flow generated by a matched job paying $w$. Because an employed worker quits when aware of a higher outside offer, the employer’s value of a continuing match solves the asset pricing equation:

$$rJ(w) = p - w - \{\delta + \lambda s(w)[1 - F(w)]\} J(w)$$

Putting it differently,

$$J(w) = \frac{p - w}{r + d(w)}$$
$$d(w) = \delta + \lambda s(w)[1 - F(w)]$$

where $d(w)$ denotes the total job separation rate.

An employer’s expected profit flow per worker contacted is the product of the hiring probability per worker contacted and the value of filling a job vacancy, i.e.,

$$\pi(w) = \max_w h(w)J(w)$$

Optimal equilibrium wage offers must satisfy the first order condition:

$$\frac{h'(w)}{h(w)} = \frac{1}{p - w} + \frac{d'(w)}{r + d(w)}$$

Equivalently,

$$\frac{\lambda s(w)F'(w)}{\delta + \lambda s(w)[1 - F(w)]} + \frac{\lambda s(w)F'(w) - \lambda s'(w)[1 - F(w)]}{r + \delta + \lambda s(w)[1 - F(w)]} = \frac{1}{p - w} \quad (1.15)$$

Easy to show that the second order condition is also satisfied.
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Figure 1.5: Equilibrium wage distributions (endogenous search effort)

Illustrative calibration: $\lambda = 0.3$, $\delta = 0.1$, $p = 3$, $b = 1$

1.5.1.3 Discussion

Following Mortensen [2002b], the cost function of search effort is assumed to be of the form:

$$c(s) = \frac{s^2}{2}$$

Figure 1.5 plots the equilibrium distributions of wage offers and earnings, predicted by the model.

Again as in the basic Burdett-Mortensen [1998] model, both the earnings and offers densities are strictly increasing and convex when search effort is endogenous. It is obvious that this generalization of the model also fails to explain the shapes of wage dispersion.

Compared with the basic Burdett-Mortensen [1998] model, this model doesn’t change much. Indeed, there is also a negative relationship between wage offers and quit rates across employers, i.e.,

$$\frac{\partial \lambda s(w)[1 - F(w)]}{\partial w} < 0$$

from $s'(w) < 0$. Further, the assumption of endogenous search effort even induces a stronger decrease of the probability of job-to-job transition when the wage increases. This gives firms additional incentive to post high wages. Therefore as in the basic Burdett-Mortensen [1998] model, the equilibrium wage density is again monotonically increasing in wages.
1.5.2 Investment in match-specific human capital

1.5.2.1 Introduction

In most equilibrium search literature, job offer arrival rates have been taken to be exogenous. The last section considered the case in which offers arrival is proportional to search effort of the worker. In turn, Mortensen[1998] integrates the matching model into the equilibrium search and wage-posting framework. By this way, the demand for labor, hence offer arrival rates, are also endogenized.

Furthermore, Mortensen[1998] extends the model by introducing endogenous firm-specific investments in human capital, that is, training. The intuition of the model is the following. Search frictions imply that there is a trade-off between wages and labor turnover from the perspective of the firm. A higher wage will reduce the probability that an employee accepts job offers from other firms. The negative association between wage and labor turnover creates incentives for training employees—thus, increasing firm-specific productivity—and paying a higher wage, since the expected duration of the match is longer and the period in which the firm can recoup its investment thus increases. Consequently, the model predicts a positive relation between the amount of training supplied and the wage paid. Firms offer wage packages consisting of both a wage and a training contract. In equilibrium, firms choose different levels of training and wage offers, which results in endogenous within-market productivity differences and, consequently, a dispersed equilibrium wage offer distribution.

1.5.2.2 Model

Matching function All workers and firms are respectively identical ex ante. The labor force, the measure of which is normalized to 1, consists of \( u \) unemployed and \( 1 - u \) employed workers. Firms post wages and the workers search, both as unemployed and employed.

We refer to \( v \) the vacancy rate. For simplicity, we assume that employed and unemployed workers are perfect substitutes in the matching process and all search at unity intensity, i.e., only the number of searching workers matter.

The matching function, measuring the number of encounters between
workers and firms, is a function of the number of vacancies, \( v \), and the number of unemployed and employed workers, \( u \) and \( 1-u \). Formally,

\[
m(v, u, 1-u) = m(v, 1)
\]

This function is assumed increasing in both its arguments, concave and homogeneous of degree 1.

The job vacancies and workers that are matched at any point in time are randomly selected from the set \((v, 1)\). Hence the processus that changes the state of vacant jobs is Poisson with rate

\[
q(v) = \frac{m(v, 1)}{v}
\]

Clearly, the rate at which vacancies become filled is decreasing with the number of vacancies. On the other side, workers receive job offers according to a related Poisson processus with rate

\[
\lambda(v) = \frac{m(v, 1)}{1}
\]

where \( \lambda(v) \) is an increasing concave function.

**Labor market flows**  Also assume that job destruction occurs at Poisson rate \( \delta \). Worker search behavior and labor turnover flows in this model are the same as described in the basic Burdett-Mortensen[1998] model. We simply restate here the equilibrium conditions:

\[
u = \frac{\delta}{\delta + \lambda(v)}
\]

\[
G(w) = \frac{\delta F(w)}{\delta + \lambda(v)(1 - F(w))}
\]

**Human capital investment and vacancy setting**  Let \( k \) represent match specific investment per worker, and let the value of worker productivity be an increasing concave function of this investment denoted as \( pf(k) \). Because investments of this form are made after worker and firm meet, and future return are lost once worker and job separate, the asset value of a vacant job solves:

\[
rV = -c + q(v)[u + (1-u)G(w)](J(k) - k - V)
\]
where $c > 0$ is the flow cost of recruiting per vacancy, $q(v)$ is the rate at which workers are contacted per vacancy, $[u + (1 - u)G(w)]$ is the probability that the worker contacted will accept an offer $w$, $(J(k) - k - V)$ is the net capital gain associated with converting a vacancy job to a filled one. In other words, the expected net return to holding a job vacant, the left side of the equation, equals the expected net return to filling a vacancy, the right side of the equation.

As free entry eliminates pure profit in vacancy creation, $V = 0$, the above asset pricing equation implies

$$J(k) = \frac{c}{q(v)[u + (1 - u)G(w)]} + k = \frac{c(\delta + \lambda(v)(1 - F(w)))}{q(v)\delta} + k$$

(1.16)

Because employed workers quit when they receive a higher alternative wage offer, the expected present value of the employer’s future flows of quasi-rents once a worker is hired at wage $w$, $J(w)$, solves:

$$rJ(k) = pf(k) - w - \lambda(v)[1 - F(w)][J(k) - V] - \delta J(k)$$

Equivalently,

$$J(k) = \frac{pf(k) - w}{r + d(w)}$$

(1.17)

$$d(w) = \delta + \lambda(v)[1 - F(w)]$$

where $d(w)$ denotes the total match separation rate.

Finally, the equilibrium number of vacancies in the market is jointly determined by equations (1.16) and (1.17),

$$\frac{cv}{\lambda(v)} = \left(\frac{\delta}{d(w)}\right) \left(\frac{pf(k) - w - k(r + d(w))}{r + d(w)}\right)$$

(1.18)

In other words, the expected cost of filling a vacancy, the left side, equals the net expected present value of the future profit attributable to filling one, the right side.

Given $\lambda(v)$ increasing and concave and the Inada conditions $\lambda(0) = 0$ and $\lambda'(0) = \infty$, exactly two solutions exist to the equation (1.18), the first at $v = 0$, and the second at some strictly positive number. Only the positive solution is stable in the sense that a simple entry process starting with positive vacancies will find the positive equilibrium.
Investment and wage policies of the firm  An employer’s expected profit flow per worker contacted is the product of the hiring probability per worker contacted, \( h(w) \), and the value of filling a job vacancy net of match specific investment, \( J(k) - k \). Namely,

\[
\pi = \max_{k,w} h(w)(J(k) - k)
\]

where

\[
h(w) = \frac{u + (1 - u)G(w)}{u + (1 - u)} = \frac{\delta}{\delta + \lambda(v)(1 - F(w))}
\]

For any choice of wage offer \( w \), the optimal investment policy is fully characterized by the first order condition:

\[
pf'(k) = r + \delta + \lambda(v)(1 - F(w))
\]

given the standard production function assumptions: \( f(0) = 0 \), \( f'(0) = \infty \), \( f'(k) > 0 \) and \( f''(k) < 0 \). Namely, the marginal return on employer investment in match specific training must equal the relevant time rate of discount, \( i.e. \), the interest rate plus the match separation rate. As an implication, employers who offer higher wages invest more in match specific capital, \( i.e. \)

\[
k'(w) = \frac{-\lambda(v)F'(w)}{pf''(k)} > 0
\]

because workers quit more frequently employers who offer lower wage. Hence, workers employed at higher wages are more productive even though all workers and firms are identical ex ante.

Next, consider the wage policy of the firm. To simplify the notation, define \( \kappa(w) \equiv pf(k) - w - (r + d(w))k \). The optimal wage policy of the firm is characterized by a wage choice that maximizes the employer’s total expected profit flow per worker contacted. Formally,

\[
w = \arg \max h(w) \frac{\kappa(w)}{r + d(w)}
\]

Optimal wage offers must satisfy the first order condition:

\[
\frac{h'(w)}{h(w)} = \frac{-\kappa'(w)}{\kappa(w)} + \frac{d'(w)}{r + d(w)}
\]
by the envelop theorem. Equivalently,
\[
\frac{\lambda(v)F'(w)}{d(w)} = \frac{1 - k(w)\lambda(v)F'(w)}{\kappa(w)} = \frac{\lambda(v)F'(w)}{r + d(w)}
\]
Rearranging the terms and making use of the relation \(\frac{\kappa(w)}{d(w)(r + d(w))} = \frac{c}{q(v)}\), one obtains the following differential equation that fully characterizes the firm’s optimal wage policy:
\[
\lambda(v)F'(w) = \frac{1}{k(w) + \frac{c}{q(v)}[r + 2d(w)]}
\]  
(1.20)
associated with the boundary condition \(F(w) = 0\) where the lower bound of the wage support is the reservation wage of the unemployed, \(b\).

**Function specification and equilibrium solution** In line with Pis sarides[2000], the matching function \(m(\cdot)\) is specified to be of the form of a Cobb-Douglas production function:
\[
m(v, 1) = v^\alpha 1^{1-\alpha} = v^\alpha
\]
where \(\alpha \in (0, 1)\). Therefore, the offer arrival rate is \(\lambda(v) = v^\alpha\) and the rate at which a vacancy is filled is \(q(v) = v^{\alpha-1}\).

Mortensen[1998] finds that the model allows for a uni-modal wage density in equilibrium if the production function, \(f(k)\), is of the form:
\[
f(k) = k^\gamma
\]
where \(\gamma \in (0, 1)\). In this case, the optimal investment of the firm, \(k(w)\), is bounded in the lower limit by:
\[
pk(b)^\gamma = p
\]
where \(p\) is the observed lowest productivity\(^7\).

Given the forms of the function, labor market equilibrium is solved by the following ODE system:
\[
F'(w) = \frac{1}{v^\alpha k(w) + \frac{\alpha}{\gamma}[r + 2(\delta + v^\alpha(1 - F(w)))]
\]
\[
k'(w) = \frac{-v^\alpha F'(w)}{p\gamma(\gamma - 1)k(w)^{\gamma-2}}
\]
\(^7\)Another way to fix the boundary condition is to set \(p = 1\), so that output per worker in the least productivity job is normalized to be unity.
defined on the support \( w \in [b, \bar{w}] \) and associated with the boundary conditions:

\[
F(w) = 0 \\
k(w) = \left( \frac{p}{\bar{w}} \right)^{\frac{1}{\gamma}}
\]

where the equilibrium number of vacancies, \( v \), is determined by:

\[
\frac{c}{v^{\alpha-1}} - \left( \frac{pk(b)^{\gamma} - b - k(b)(r + \delta + v^{\alpha})}{r + \delta + v^{\alpha}} \right) \left( \frac{\delta}{\delta + v^{\alpha}} \right) = 0
\]

1.5.2.3 Discussion

Figure 1.6 plots the equilibrium distributions of wage offers and earnings, predicted by the model. Contrarily to the basic Burdett-Mortensen[1998] model, the equilibrium earnings density could be of uni-modal form when match specific investment is included.

Indeed, because investment in human capital implies a cost to the firm whereas its return to scale decreases with the amount of investment \((0 < \gamma < 1)\), the firm is discouraged from posting more high-wage jobs. Therefore, a hump-shaped (or decreasing) wage density is well allowed in equilibrium. Putting it differently, differentiating equation (1.20) with respect to \( w \) gives:

\[
\frac{F''(w)}{F'(w)} = -\left[ k'(w) + \left( \frac{2c}{\delta(q(v))} \right) d'(w) \right] \\
\quad \quad \quad = (\lambda(v)F'(w))^2 \left[ \frac{2c}{\delta(q(v))} + \frac{1}{pf''(k(w))} \right]
\]

by use of equation (1.19). If the effect of decreasing return to scale is sufficiently strong, a raise of wage will induce a decrease of \( F'(w) \), i.e., \( F''(w) < 0 \) at least for some high wage rates. Consequently, a hump shape (or decreasing shape) of wage density \((g(w))\) is well allowed by the model.

Rosholm-Svarer[2004] extend the Mortensen[1998] model to allow for different offer arrival rates in employment and unemployment. The extension allows for extra flexibility when the model is confronted with real data. Rosholm-Svarer[2004] subsequently estimate the parameters of the model on
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Figure 1.6: Equilibrium wage distributions (match-specific investment)

Illustrative calibration: $\delta = 0.1, b = 1, \alpha = 0.75, \gamma = 0.6, c = 0.5, p = 6, p = 5.5$

Danish labor market data (CLS 1981-1990) using a structural nonparametric estimation procedure in line with Bontemps-Robin-Van den Berg[2000]. Rosholm-Svarer[2004] show that, generally speaking, the model provides a good characterization of some empirical features of the labor market.

Their main findings are the following. First, they find $\lambda_0 > \lambda_1 > \delta$ for all groups in study, just as in Bontemps-Robin-Van den Berg[2000]. Rosholm-Svarer[2004] recover the distributions of human capital investments and match products nonparametrically, and find that human capital investments correspond to a few (3-4) months’ earnings for high wage individuals. The relation between wages and human capital is increasing by assumption, and in addition it turns out to be convex. Rosholm-Svarer[2004] also find that the estimated unemployment rate is significantly lower than the actual steady-state unemployment rate. However, it appears that the mean expected job duration only slightly exceeds the actual mean job duration for nearly all strata. In conclusion, Rosholm-Svarer[2004] find that the model allowing for different offer arrival rates in employment and unemployment provides a good characterization of many interesting features of the labor market, especially regarding the connection between human capital (on-the-job training), wages, and job turnover.
1.5.3 Investment in general physical capital

1.5.3.1 Introduction

In contrast with investment in match specific human capital, firms may also make investment in physical capital. Physical capital is generally viewed as embodied in the job rather than the match. When the job is created, investments are made in the equipment that any worker needs to be productive in the job. Unlike the case of match specific human capital, the equipment does not vanish if the worker leaves. Instead, the employer can replace the worker by matching another with the original equipment.

Mortensen[2002a] and Robin-Roux[2002] first extend the basic Burdett-Mortensen[1998] model along this direction. In these models, the wage paid by an employer is important as a determinant of the utilization rate of the job’s equipment. Specially, capital utilization is higher for employers who pay more because they fill vacant jobs more rapidly and retain workers longer. Since the return to capital investment is increasing in the utilization rate, employers who pay more have an incentive to invest more per job in the form of general physical capital. This section presents the Mortensen[2002a] model. In addition, we allow for endogenous labor demand and job arrival rates. The Robin-Roux[2002] model is discussed at the end of the section.

1.5.3.2 Model

Matching function and Labor market flows All workers and firms are respectively identical ex ante. The unemployed searches for job offers whereas the employed searches for wage increases. Offers arrive at rate \( \lambda(v) \) when both employed and unemployed, and the rate at which a vacancy is filled is \( q(v) \). Also assume that job destruction occurs at Poisson rate \( \delta \).

Worker search behavior and labor turnover flows in this model are the same as described in the basic Burdett-Mortensen[1998] model. We simply restate here the equilibrium conditions:

\[
\begin{align*}
    u & = \frac{\delta}{\delta + \lambda(v)} \\
    G(w) & = \frac{\delta F(w)}{\delta + \lambda(v)(1 - F(w))}
\end{align*}
\]
Physical capital investment and asset pricing  Let $k$ represent physical capital investment embedded in the job, and let the value of worker productivity be an increasing concave function of this investment denoted as $pf(k)$. Because any job can be rematched with another worker after a quit but is destroyed at rate $\delta$, the value of a job when filled conditional on the investment made in general capital $k$ solves:

$$rJ(k) = pf(k) - w - \lambda(v)[1 - F(w)][J(k) - V(k)] - \delta J(k)$$

The value of the job when vacant, denoted as $V(k)$, is determined by the asset pricing equation:

$$rV(k) = -c + q(v)[u + (1 - u)G(w)][J(k) - V(k)] - \delta V(k)$$

where $c > 0$ is the recruitment cost per vacancy, $q(v)$ the rate at which workers are contacted per vacancy, $[u + (1 - u)G(w)]$ the probability that the worker contacted will accept an offer $w$, $[J(k) - V(k)]$ the capital gain associated with converting a vacancy job to a filled one, and $\delta$ the rate at which a vacancy is destroyed.

By subtracting corresponding sides of equation $rV(k)$ from equation $rJ(k)$, one obtains:

$$J(k) - V(k) = \frac{pf(k) - w + c}{r + d(w) + Q(w)}$$

where

$$d(w) = \delta + \lambda(v)[1 - F(w)]$$
$$Q(w) = q(v)[u + (1 - u)G(w)]$$

Note that $d(w)$ represents the total match separation rate and $Q(w)$ represents the rate at which a vacancy is filled conditionally on the offer $w$.

Substituting once again equation $J(k) - V(k)$ into equation $rV(k)$ gives:

$$V(k) = \frac{[pf(k) - w]Q(w) - c[d(w) + r]}{(r + \delta)[r + d(w) + Q(w)]}$$

(1.21)
CHAPTER 1. EXPLANATION OF WAGE DISPERSION

Optimal capital investment policy of the firm  As physical capital is embedded in the job but does not vanish if the worker leaves, it is now the expected discounted return to the value of the job when vacant net of capital investments, \((r + \delta)[V(k) - k]\), that is the relevant profit concept. Wages and capital investments are set to maximize the profit, i.e.,

\[
\pi = \max_{k,w} (r + \delta)[V(k) - k]
\]

\[
= \max_{k,w} \left\{ \frac{[pf(k) - w]Q(w) - c[d(w) + r]}{r + d(w) + Q(w)} - (r + \delta)k \right\}
\]

Optimal capital investment policy of the firm must satisfy the first order condition:

\[
pf'(k) \left( \frac{Q(w)}{r + d(w) + Q(w)} \right) = r + \delta
\]

given the standard production function assumptions: \(f(0) = 0\), \(f'(0) = \infty\), \(f'(k) > 0\) and \(f''(k) < 0\). The rate of return on capital, the left side, is the product of the marginal productivity of capital \(pf'(k)\) and the capital utilization rate. Given that \(Q(w)\) represents the transition rate from a vacant job to a filled job, and that \(d(w)\) represents the transition rate from filled to vacant, the ratio \(\frac{Q(w)}{d(w) + Q(w)}\) is therefore the steady state fraction of time the job is filled, i.e., the capital utilization rate. This rate increases with the wage offered because \(d'(w) < 0\) and \(Q'(w) > 0\).

As the second order condition for an interior solution requires that \(pf(k)\) be strictly concave, employers who offer higher wages invest more in physical capital, i.e.,

\[
k'(w) = (r + \delta)\frac{d'(w)Q(w) - [r + d(w)]Q'(w)}{pf''(k)Q(w)^2} > 0
\]

from \(d'(w) < 0\), \(Q'(w) > 0\) and \(f''(k) < 0\).

Wage setting  Similarly, optimal wage offers of the firm satisfy the following first order condition:

\[
\frac{[pf(k) - w]Q(w) - c[d(w) + r]}{[pf(k) - w]Q(w) - c[d(w) + r]}' = \frac{[r + d(w) + Q(w)]'}{[r + d(w) + Q(w)]}
\]
by the envelop theorem. By substituting \(Q(w)\) and \(d(w)\) out of the above optimal condition and making use of the relation \(\frac{m(w)}{n(w)}' = \frac{m'(x)}{n'(x)}\) where \(m(w)\) and \(n(w)\) any two continuously differentiable functions of \(w\), one obtains:

\[
\frac{\delta q(v) - c\lambda v F'(w)[r + 2(\delta + \lambda v)(1 - F(w))]}{\delta q(v)[pf(k) - w] - c[\delta + \lambda v](1 - F(w))[r + \delta + \lambda v](1 - F(w))} = \frac{\lambda v F''(w)[r + 2(\delta + \lambda v)(1 - F(w))]}{\lambda v F''(w)[r + 2(\delta + \lambda v)(1 - F(w))]}
\]

The optimal wage policy of the firm is fully characterized by this differential equation associated with the boundary condition \(F(w) = 0\) where the lower bound of the wage support is the reservation wage of the unemployed, \(b\).

**Vacancy setting** In equilibrium, free entry eliminates pure rents from vacant jobs given embedded capital investment. Therefore the equilibrium condition for the supply of vacant jobs is \(V(k) - k = 0\), implying that:

\[
\frac{[pf(k(w)) - w]Q(w) - c[d(w) + r]}{r + d(w) + Q(w)} = (r + \delta)k(w)
\]

Because employers are all identical ex ante, optimality again requires that all wage offered yield the same profit. Hence, equilibrium number of vacancies in the market solves:

\[
\frac{(pf(k(w)) - w)\left(\frac{\delta q(v)}{\delta + \lambda v}\right) - c(r + \delta + \lambda v)}{r + \delta + \lambda v + \left(\frac{\delta q(v)}{\delta + \lambda v}\right)} = (r + \delta)k(w) \quad (1.24)
\]

### 1.5.3.3 Discussion

Robin-Roux[2002] also extend the basic Burdett-Mortensen[1998] model to allow for endogenous capital determination. In order to make their model more flexible when confronted with real data, *two-sided search*, meaning that firms must produce a specific hiring effort which is by itself costly, and *balanced matching technology*, by which firms are sampled with a probability proportional to their size, are incorporated as assumptions into the model. In particular, Robin-Roux[2002] assume that the decision for a firm to enter the labor market involves a decision about general physical capital. Ex ante
identical firms will in equilibrium choose different levels of capital. The equilibrium distribution of capital then induces an equilibrium distribution of labor productivity.

Robin-Roux[2002] then estimate their model using French accounting firm data (BIC). As in many other empirical studies, Robin-Roux[2002] find that capital does explain a significant share of the variance of logged wages, logged employment and labor share in the sample. However, the residual variance can not only be attributed to measurement errors, it remains a part left to be explained. Note, also, that their model is not able to generate distributions of wages, value-added and employment as dispersed as the true ones. The predicted standard derivations are indeed far from being similar to their empirical counterparts. Importantly, although the wage offer density, $F'(w)$, is monotonously decreasing, the earnings density, $G'(w)$, is instead monotonously increasing with wage. As pointed out by the authors: “whether this deficiency is a feature of the structural model or of the particular specification we have used for estimation and simulation, we cannot say”.

To give a quantitative illustration of the Mortensen[2002a] model, two functions should be parametrically specified: the matching technology $m(v)$ and the production function $f(k)$. For the sake of conformity, these two functions are specified to be of the same form as in the precedent model.
1.5. ENDOGENOUS HETEROGENEITY

where firms make investment in match specific human capital:

\[ m(v) = v^\alpha \]
\[ f(k) = k^\gamma \]

where \( \alpha, \gamma \in (0, 1) \). Figure 1.7 plots the equilibrium distributions of wage offers and earnings, predicted by the model.

Just as in Robin-Roux[2002], although the offer density decreases with wage, experimentation suggests that the earnings density is always increasing for all realistic combinations of parameter value. These results suggest that it may be difficult to reconcile this version of the model with data on the distributions of wages offered and paid.

One possible explanation to the increasing shape of earnings density is that investment in general physical capital affects less strongly the tradeoff of the firm than investment in human capital. In case of investment in human capital, the firm faces the tradeoff between instantaneous profit and job duration, and sets the wage to maximize the expected discounted profit flow of a filled job, \( w = \arg \max h(w)(J(k) - k) \). Because investment in human capital implies a cost to the firm whereas its return to scale decreases with the amount of investment, the firm is discouraged from posting more high-wage jobs.

However in case of investment in general physical capital, the equipment does not vanish if the worker leaves. It is now the expected discounted return to the value of the job when vacant that is the relevant profit concept. Wages are set to maximize the value of the job when vacant, \( w = \arg \max (r + \delta)[V(k) - k] \). Therefore, investment in physical capital affects less the tradeoff of the firm between instantaneous profit and job duration, so that firms are less discouraged from posting more high-wage jobs. Consequently, the density of offers, \( F'(w) \), decreases less rapidly with wage(see figures 1.7 and 1.6). Indeed, because

\[ G'(w) = \frac{\delta(\delta + \lambda(v))F'(w)}{(\delta + \lambda(v)(1 - F(w)))^2} \]

\[ G''(w) = \frac{\delta(\delta + \lambda(v)) [F''(w)(\delta + \lambda(v)(1 - F(w))) + 2\lambda(v)(F'(w))^2]}{(\delta + \lambda(v)(1 - F(w)))^3} \]
it could be that the earnings density, $G'(w)$, is monotonously increasing with wage even though the wage offer density, $F''(w)$, is monotonously decreasing with wage.

1.6 Intra-firm wage dispersion and wage growth

1.6.1 Between-firms Bertrand competition, counter-offer and sequential auctions

1.6.1.1 Introduction

In most equilibrium search models, wages are invariant over the duration of a job match by assumption. This characteristic contradicts several empirical evidences. For example, within-firm wage growth is widely observed, employment contracts that condition pay on length of service with an employer are common. In addition, the estimated coefficient on a job-tenure variable in a standard wage equation is always positive.

However, a purely theoretical critique of the flat wage-tenure profile specification also exists. Namely, if an employer has monopsony market power, then she suffers a loss of future rents when an employee quits to take another job. Rather than accept the loss, the employer will have an incentive to retain the employee. Putting it differently, there is no reason why the employer should let the employee leave the firm as such passive behavior is clearly sub-optimal. In short, a quit is not a jointly efficient separation in most equilibrium search models.

From these viewpoints, Postel-Vinay and Robin[2002] incorporate the retention policy of the firm into the Burdett-Mortensen[1998] model in order to explain wage growth and wage dispersion within the firm. In Postel-Vinay and Robin[2002], employers are assumed to make counter-offers when an employee receives an outside offer. All employers pay workers hired from unemployment their reservation wage, but compete with any alternative employer that the worker meets subsequently. When an employed worker has an outside employment opportunity, the employer offering the alternative job and the worker’s current employer bid against each other until the expected future value of either hiring or keeping the worker is zero for one of the two.
1.6. **INTRA-FIRM WAGE DISPERSION**

If the productivity of the worker with the current employer exceeds that with the alternative, the worker stays but is paid a wage equal to his productivity in the alternative job. Otherwise, the worker quits to take the outside offer at a wage that equalizes the value of continued employment with the value of alternative employment. In such case, the separation is efficient, and the worker’s subsequent wage is determined by the outcome of a subsequent auction involving the current employer and a prospective alternative. By this way, the model permits to explain not only wage-tenure effect but also job-to-job mobilities that result in wage cuts.

Wage dispersion exists in Postel-Vinay and Robin[2002] both within and across firms. Indeed, wages are positively but not perfectly correlated with both job tenure and labor market experience as measured by the duration of the worker’s current employment spell. Because the higher wage can mostly be obtained once employed\(^8\), an unemployed worker is willing to accept and will be offered a wage below the opportunity cost of employment. Once employed, the worker either receives wage raises or moves to a more productive employer. Although the worker is generally willing to take a pay cut to make such a move, that willingness is justified by the expectations that the new employer will make even larger counter-offers in the future. Hence, this approach provides a rich theory of both inter- and intra-firm wage differences. The essence of this model follows.

1.6.1.2 **Model**

**Search behavior of the worker** All workers are identical ex ante. In contrast, assume that firms differ by an exogenous technological parameter \(p\), with distribution \(\Gamma\) across firms over the support \(p \in [p_{\text{min}}, p_{\text{max}}]\). This distribution is assumed continuous with density \(\gamma\).

Let \(U\) and \(W(w|p)\) denote the expected discounted lifetime income of the unemployed and the worker employed at wage \(w\) by a firm with productivity \(p\), respectively. All workers search, unemployed workers search for job offers whereas employees search for wage increases. Offers arrive at rate \(\lambda\) when both unemployed and employed. Because every employer pays the unem-

\(^8\)This is due to an *employment effect*, see section 1.5.1 for explanation.
ployed his reservation wage, denoted by $R_0(p)$, the expected lifetime income of an unemployed worker solves:

$$rU = b + \lambda \int_{p}^{\overline{p}} [W(R_0(x)|x) - U] d\Gamma(x) = b$$

where $b$ is the unemployment compensation.

Now consider the employee. When a type-$p$ firm’s employee receives an outside offer from a type-$p'$ firm, both firms enter a Bertrand competition won by the most productive firm. Let $R(p_1, p_2)$ represent the employee’s reservation wage where $p_1 \leq p_2$. Three cases should be distinguished. First, since the current employer will respond by paying the worker up to the worker’s productivity $p$, but not more, the alternative employer with a productivity $p' \in (p, \overline{p}]$ could offer a wage greater than $R(p, p')$ where $W(R(p, p')|p') = W(p|p)$, to attract the worker. If so, the worker accepts the outside offer and goes working at the type-$p'$ firm for a wage $R(p, p')$. Next, consider the case $W(w|p) = W(q(w|p)|q(w|p))$ where $q(w|p)$ is called the reservation productivity. If the offer is from a competitive firm with a productivity $p' \in [p, q(w|p))$, then the challenging firm is obviously less attractive to the worker than his current employer since it cannot even offer him his reservation wage. The worker thus rejects the offer and continues his current employment relationship at an unchanged wage rate. Last, if the offer stems from a challenging firm with a productivity $p' \in [q(w|p), \overline{p}]$, then the offer is matched by the current employer, in which case the challenging firm will not be able to attract the worker but the current employer will have to grant the worker a wage increase up to $R(p', p)$ where $W(R(p', p)|p) = W(p'|p')$, to retain him from accepting the challenging firm’s offer. So formally, the expected lifetime incoming of an employed worker solves the asset pricing equation:

$$rW(w|p) = w - \delta [W(w|p) - U] + \lambda \int_{p}^{\overline{p}} [W(p|p) - W(w|p)] d\Gamma(x) + \lambda \int_{q(w|p)}^{p} [W(x|x) - W(w|p)] d\Gamma(x)$$

where $\delta$ is the job destruction rate.
Making use of the relation $W(w|p) = W(q(w|p)|q(w|p))$, we rewrite the above equation as:

\[ rW(w|p) = w - \delta [W(w|p) - U] + \lambda [1 - \Gamma(p)] [W(p|p) - W(w|p)] \\
- \lambda [\Gamma(p) - \Gamma(q(w|p))]W(w|p) + \int_{q(w|p)}^{p} W(x|x)d\Gamma(x) \]

By substituting once again $W(x|x) = \frac{x + \delta U}{r + \delta}$ into the above equation, one obtains:

\[ (r + \delta)W(w|p) = w + \delta U + \frac{\lambda}{r + \delta} \int_{q(w|p)}^{p} (1 - \Gamma(x))dx \]  

(1.25)

by an integration by parts.

**Determination of reservation productivity and reservation wage**

First let’s consider the determination of the reservation productivity $q(w|p)$. By substituting $W(w|p) = W(q(w|p)|q(w|p)) = \frac{q(w|p) + \delta U}{r + \delta}$ into equation (1.25), one obtains:

\[ q(w|p) = w + \lambda \frac{\sqrt{r + \delta}}{r + \delta} \int_{q(w|p)}^{p} (1 - \Gamma(x))dx \]

Note that $q(w|p) \geq w$. Differentiating this equation with respect to $w$ gives:

\[ q'(w|p) = \frac{1}{1 + \frac{\lambda}{r + \delta}(1 - \Gamma(q(w|p)))} \]  

(1.26)

Known the productivity of the current employer $p$, the reservation productivity is fully characterized by this differential equation associated with the boundary condition $q(p|p) = p$.

Now consider the reservation wage of an employee $R(p_1, p_2)$ where $p_1 \leq p_2$ are the productivity of the two firms in competition no matter which one the employee is currently in. Substituting $w = R(p_1, p_2)$ into equation $q(w|p)$ gives:

\[ q(R(p_1, p_2)|p_2) = R(p_1, p_2) + \lambda \frac{\sqrt{r + \delta}}{r + \delta} \int_{q(R(p_1, p_2)|p_2)}^{p_2} (1 - \Gamma(x))dx \]

As $q(R(p', p)|p) = p'$, the above relation implies:

\[ R(p_1, p_2) = p_1 - \frac{\lambda}{r + \delta} \int_{p_1}^{p_2} (1 - \Gamma(x))dx \]  

(1.27)
Equation (1.27) indicates that, when the productivity of the challenging firm is strictly greater than that of the current employer, the employee moves to the more productive firm with a reservation wage less than the productivity of his current employer.

Similarly, by substituting \( U = W \left( R_0(p) \right) \) into equation (1.25) and making use of the relation \( rU = b \), one obtains:

\[
R_0(p) = b - \frac{\lambda}{r + \delta} \int_{q_0}^{p} (1 - \Gamma(x))dx
\]

(1.28)

where \( q_0 \equiv q(R_0(p)) \) denotes the reservation productivity to recruit an unemployed worker. Again, equation (1.28) indicates that the unemployed worker is willing to accept a reservation wage below the opportunity costs of employment. Note that from \( U = W(\left. R_0(p) \right| p) = W(q_0|q_0) \), \( W(q_0|q_0) = \frac{q_0 + \delta U}{r + \delta} \) and \( rU = b \), easy to see

\[
q_0 = b = p
\]

The reservation productivity to recruit an unemployed worker, or the lowest productivity in economics, is just the opportunity cost of employment.

**Labor market flows and inter- and intra-firm wage dispersion** As the transition rate from unemployment to employment is \( \lambda \), and from employment to unemployment is \( \delta \), the steady state unemployment rate is again:

\[
u = \frac{\delta}{\delta + \lambda}
\]

Let \( l(p) \) be the fraction of workers in firms with a productivity \( p \), and \( L(p) \) the fraction of workers in firms with a productivity no greater than \( p \). Let also \( G(w|p) \) denote the distribution of earnings equal or less than \( w \) in type-\( p \) firms. Thus, the stock of employed workers earning \( w \) or less in firms with a productivity \( p \) is \( (1-u)l(p)G(w|p) \). Workers leave this stock because they are laid off (at rate \( \delta \)), or because they receive an offer from a challenging firm with a productivity \( p' \geq q(w|p) \) which either grants them a wage increase or induces them to leave their current firm (at rate \( \lambda [1 - \Gamma(q(w|p))]) \). On the inflow side, workers entering this stock also come from two distinct sources. Either they are hired from firms less productive than \( q(w|p) \), or they come
from unemployment. The steady-state equality between flows into and out of the stock \((1-u)l(p)G(w|p)\) thus takes the form:

\[
(1-u)l(p)G(w|p)\{\delta + \lambda(1-\Gamma(q(w|p)))\} = \gamma(p)\lambda\{u + (1-u)L(q(w|p))\}
\]

(1.29)

where \(\gamma(p)\) is the fraction of firms with a productivity \(p\).

To simplify the mathematical expression, let’s define \(\Gamma(p) = 1 - \Gamma(q(w|p))\) and \(k = \frac{1}{\delta}\). When \(w = p\), the above flow condition yields:

\[
l(p)[1 + k\Gamma(p)] = [1 + kL(p)]\gamma(p)
\]

This is a first-order differential equation of type \(\frac{dy}{dx} + P(x)y = Q(x)\) where \(P(x)\) and \(Q(x)\) are any two continuously differentiable functions of \(x\). Making use of the boundary condition \(L(p) = 0\) and solving this differential equation, one obtains:

\[
L(p) = \frac{\Gamma(p)}{1 + k\Gamma(p)}
\]

\[
l(p) = \frac{1 + k}{[1 + k\Gamma(p)]^2}\gamma(p)
\]

Next, by substituting \(L(p)\) and \(l(p)\) out of equation (1.29), one obtains:

\[
G(w|p) = \frac{[1 + kL(q)]\gamma(p)}{[1 + k\Gamma(q(w|p))]l(p)} = \left(\frac{1 + k\Gamma(p)}{1 + k\Gamma(q(w|p))}\right)^2
\]

(1.30)

where \(q(w|p)\) is the solution of equation (1.26). In Postel-Vinay and Robin [2002], \(G(w|p)\) represents intra-firm wage distribution conditional on the productivity of the firm \(p\). Its counterpart, the inter-firm wage distribution, is defined by:

\[
G(w) = \int_b^\Gamma G(w|p)d\Gamma(p)
\]

**Decomposition of wage variance** In a bivariate joint distribution of \((w, p)\), decomposition of variance means that:

\[
V(w) = V_p[E(w|p)] + E_p[V(w|p)]
\]
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The notation $V_p(\cdot)$ indicates the variance over the distribution of $p$. From the statistical viewpoint, this equation states that in a bivariate distribution, the variance of $w$ decomposes into the variance of the conditional mean function plus the expected variance around the conditional mean. More specially, the variance decomposition result implies that in a bivariate distribution, variation in $w$ arises from two sources: 1) regression variance, denoted by $V_p[E(w|p)]$, is the variation because $E[w|p]$ varies with $p$, and 2) residual variance, denoted by $E_p[V(w|p)]$, is the variation because, in each conditional distribution, $w$ varies around the conditional mean. Therefore,

$$\text{total variance} = \text{regression variance} + \text{residual variance}$$

Now return to the model and let’s first consider the case of homogenous workers. In Postel-Vinay and Robin[2002], the first term $V_p[E(w|p)]$ is the between-firm wage variance. It reflects the fact that some firms pay higher wages on average and thus contribute to individual wage dispersion. Hence, it is quite natural to label this term the “firm effect”. The second term $E_p[V(w|p)]$ is the share of the within-firm wage variance. Its origin is identified in the model: the reason why two identical workers working at identical firms can earn different wages is that the two workers had different draws of alternative wage offers. This particular source of wage dispersion is therefore the fact that firms compete to attract workers on a frictional labor market. Hence the name “effect of market frictions”. Formally,

$$\text{wage variance} = \text{firm effect} + \text{friction effect}$$

Next consider the case of heterogenous workers. Let $\varepsilon$ denote the ability of the individual. Now the decomposition of wage variance becomes:

$$V(w) = V_p[E(w|p)] + (E_p[V(w|p)] - V(\varepsilon)) + V(\varepsilon)$$

The second term $(E_p[V(w|p)] - V(\varepsilon))$ becomes the share of the within-firm wage variance which is not due to dispersion in individual ability. The third term $V(\varepsilon)$ in this decomposition is obviously interpreted as the contribution of dispersion in unobserved individual ability. Postel-Vinay and Robin[2002] therefore refer to it as the “person effect”. Namely,

$$\text{wage variance} = \text{firm effect} + \text{friction effect} + \text{person effect}$$
1.6. INTRA-FIRM WAGE DISPERSION

1.6.1.3 Discussion

In line with Bontemps-Robin-Van den Berg[2000], we assume $\Gamma(p)$ follows a Pareto distribution:

$$\Gamma(p) = 1 - (p/p_0)^{\beta}$$

where $p_0 = b$. Figure 1.8 plots the intra-firm distributions of wage offers and earnings, predicted by the model.

Postel-Vinay and Robin[2002] structurally estimate the model using French panel data (DADS 1996-1998). They find that the model generally provides a good fit to the actual wage data. In particular, allowing for individual heterogeneity improves the fit a lot at the expense of analytical complexity. Postel-Vinay and Robin[2002] then use this structural estimation to provide a decomposition of cross-employee wage variance. They find that the share of the cross-sectional wage variance that is explained by person effects varies across skill groups. Specifically, this share lies close to 40% for high-skilled white collars, and quickly decreases to 0% as the observed skill level decreases. The contribution of market imperfections to wage dispersion is typically around 60%.

The contribution of Postel-Vinay and Robin[2002] is rather attractive. Also, their work deserves some further remarks. First, wage dispersion in Postel-Vinay and Robin[2002] depends on productivity dispersion. If instead productivity dispersion degenerates, all employers pay workers hired from unemployment the opportunity costs of employment; when an employed worker has an outside employment opportunity, the competition between the current employer and the challenging employer brings the bid towards the competitive wage (see equations (1.27) and (1.28)). Note, particularly, that these results are true even though the employer has monopsony market power. Therefore, equilibrium wage offers are degenerated to two points: one at the unemployment compensation and another at the marginal product. In short, the model of Postel-Vinay and Robin[2002] cannot explain pure wage dispersion.

In addition, Van den Berg-Van Vuuren[2003] and Bontemps-Robin-Van den Berg[2000] all indicate the pure friction effect typically explain no more than 10% of wage variation in the data. It is rather the interaction between
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Figure 1.8: Equilibrium wage distributions (Postel-Vinay and Robin[2002])

Illustrative calibration: $\lambda = 0.3, \delta = 0.1, b = 1, \beta = 3, p = 1, \bar{p} = 6, \underline{p} = 5.5$

productivity variation and frictions that provides a good fit to within-market wage distributions. Consequently, the 60% share of the “friction effect” in Postel-Vinay and Robin[2002] is rather attributed to an interaction effect between market friction and firm/worker heterogeneity.

1.6.2 Wage-Tenure Contract Posting

1.6.2.1 Introduction

Just as argued by Postel-Vinay and Robin[2002], when an employee receives an outside offer of a wage greater than her current wage but lower than her marginal productivity, there is no reason why his current employer should let him leave the firm as such passive behavior is clearly sub-optimal. In Postel-Vinay and Robin[2002], when an employed worker has an outside employment opportunity, the employer offering the alternative job and the worker’s current employer bid against each other until the expected future value of either hiring or keeping the worker is zero for one of the two. Consequently, a quit separation is always jointly efficient.

Along another line, Stevens[2002] shows that a firm need not to respond to outside offers received by any of its employees. Instead, the firm could offer wage-tenure contracts to control inefficient quit turnover of her labor force. Stevens[2002] assumes that all workers and firms are risk neutral.
With complete financial markets, the first best contract specifies a hiring fee that workers pay on being hired. The worker is then paid marginal product while employed. Contrarily if workers are credit constrained, a two-tier wage-tenure contract is optimal: a firm pays a low but positive wage for short tenures, and pays marginal product once the worker’s tenure exceeds some threshold. Stevens[2002] also establishes that equilibrium in this case is always degenerate, where all firms make strictly positive profits, offer the same initial wage and there is no quit turnover. Furthermore in this degenerate equilibrium, there is a continuum of alternative contracts that are payoff equivalent to the equilibrium step-contracts. In sum, as the firm in Stevens[2002] has effectively ruled out inefficient quit turnover of the employee and consequently extracts a higher profit per match, the equilibrium of Burdett-Mortensen[1998] is sub-optimal.

Therefore, the argument that more efficient wage contracts potentially exist whereas a fixed-wage contract is sub-optimal represents a serious challenge to the Burdett-Mortensen[1998] model. Nevertheless, the key characteristics of the Stevens[2002] model—degenerate offers and no inefficient separation—are very sensitive to the assumption that workers are risk neutral. Burdett-Coles[2004] show that a little bit of risk aversion restores a non-degenerate equilibrium where identical employers offer different contracts in equilibrium. Firms in Burdett-Coles[2004] differ from those in Burdett-Mortensen[1998] mainly in that they respond to on-the-job search behavior of the employee by modifying their wage policy. Although firms in the market make different offers in equilibrium, all post a wage-tenure contract that implies a worker’s wage increases smoothly with tenure at the firm. As firms make different offers, there is job turnover, as employed workers move jobs as the opportunity arises. This implies the increase in a worker’s wage can be due to job-to-job movements as well as wage-tenure effects. The underlying empirical implications are that within a firm, workers with shorter tenures earn lower wages and are more likely to quit, while across firms, those firms that offer more generous wage-tenure contracts have lower average quit rates and attract more workers. As a consequence, there is a non-degenerate equilibrium distribution of initial wage offers both within and across firms. Specially, Burdett-Coles[2004] show that the Stevens[2002] solution is a limiting case
to the model when again workers are assumed to be risk neutral.

Burdett-Coles[2004] assume that workers are credit constrained (imperfect capital markets) and risk averse. These assumptions imply there are two forces that determine the nature of this contract. First, as capital markets are imperfect, there is an insurance problem where, other things being equal, risk averse workers prefer a constant wage stream. Second, there is a moral hazard problem where an employee quits if a better outside offer is received. An optimal contract trades off these two competing effects where wages increase smoothly with tenure. By offering a worker a smaller wage today but a greater wage at some future date, a firm reduces its current wage bill and also increases an employee’s expected return to staying with the firm. In short, the essence of the Burdett-Coles[2004] model follows.

1.6.2.2 Model

Labor market flows  Time is continuous and only steady states are considered. All workers and firms are respectively identical ex ante. Furthermore, assume that workers are risk averse and credit constrained. Let \( u(\cdot) \) denote the instantaneous utility of income. Assume that \( u(\cdot) \) is strictly increasing, strictly concave, and twice differentiable such that \( \lim_{w \to 0^+} u(w) = -\infty \). Offers arrive at Poisson rate \( \lambda \) when unemployed and unemployed, job destruction occurs at Poisson rate \( \delta \).

Any job offer is fully described by the wage-tenure contract offered by the firm. Such a contract specifies the wage the worker receives as a function of his or her tenure at that firm. As employees are equally productive, assume a firm offers all new hires the same contract. The worker’s continuation value of a contract, denoted by \( W = W(w(t)) \) where \( w(t) = w(t)|_{t \geq 0} \) is a wage sequence indexed to tenure \( t \), is equal to the expected future stream of utility. Let \( F(W) \) represent the proportion of firms whose contract offer yields an expected lifetime utility value no greater than \( W \) where \( W \in [\underline{W}, \overline{W}] \). A contract is acceptable if its value exceeds the value of unemployment denoted by \( U \), i.e., \( \underline{W} \geq U \). Also let \( G(W) \) represent the fraction of workers employed at contract value \( W \) or less.

As the transition rate from unemployment to employment is \( \lambda \), and from employment to unemployment is \( \delta \), the steady state unemployment rate is
again

\[ u = \frac{\delta}{\delta + \lambda} \]

In steady-state, the stock of workers employed at contract value \( W \) or less is \((1-u)G(W)\). Workers leave this stock because they are laid off (at rate \( \delta \)) or because they receive a more valuable outside contract (at rate \( \lambda[1 - F(W)] \)). On the other side, unemployed workers flow into this stock because they draw a contract with initial value \( W \) or less (at rate \( \lambda F(W) \)). In steady-state, the balance between flows into and out of this stock is written as:

\[ u \lambda F(W) = (1 - u)G(W) [\delta + \lambda(1 - F(W))] \]

**Search behavior of the worker** All workers search, unemployed workers search for job offers whereas employed workers search for more valuable outside contracts. Because an unemployed worker accepts the first contract offer with value greater than \( U \), the value of unemployment solves the asset pricing equation:

\[ rU = u(b) + \lambda \int_{W}^{W'} [x - U]dF(x) \]

where \( b \) is the unemployment compensation.

However, the value of employment, \( W \), evolves over time. Because the worker’s optimal decision is to accept a more valuable outside contract whenever the opportunity arises, the value of employment solves the following continuous time Bellman equation:

\[ rW = u(w(t)) + \delta[U - W] + \lambda \int_{W}^{W'} [x - W]dF(x) + \frac{dW}{dt} \quad (1.31) \]

**Policy package of the firm: wage-tenure contract** We first consider firms’ recruitment policy. Let \( W_0 \) represent the initial value of a contract offer randomly drawn from the support \([W, W']\). Because unemployed workers accept all offers above the value of unemployment \( U \) and employed worker accept only more valuable offers, the unconditional probability that a contract with initial value \( W_0 \) will be accepted by a randomly contacted worker
CHAPTER 1. EXPLANATION OF WAGE DISPERSION

is:

\[ h(W_0) = \frac{u + (1 - u)G(W_0)}{u + (1 - u)G(W)} = \frac{\delta}{\delta + \lambda[1 - F(W_0)]} \]

Let \( p \) denote the marginal product per job-worker match, and \( J = J(w(t)) \) denote the expected present value of the future incoming flow generated by a matched job. Because an employee quits when he receives a better outside offer, the employer’s value of a continuing match solves:

\[ rJ = p - w(t) - [\delta + \lambda(1 - F(W))]J + \frac{dJ}{dt} \quad (1.32) \]

An employer’s expected profit flow per worker contacted is the product of the hiring probability per worker contacted and the value of filling a job vacancy given the initial value of the contract \( W_0 \), i.e.,

\[ \pi(W_0) = \max_{w(t)} h(W_0)J(w(t)) \]

A wage-tenure contract is a sequence of wage terms that can be enforced in the future. Formally, it is the solution to the constrained problem:

\[ w(t) = \arg \max \pi(W_0) \]

\[ = \arg \max \int_0^\infty [p - w(t)]e^{-\int_0^t [r + \delta + \lambda(1 - F(W(w(x))))]dx} dt \]

subject to

\[ W(w(0)) = W_0 \]
\[ W(w(t)) \in [W, \bar{W}] \]

Solving this dynamic optimization problem gives the following optimal condition which fully characterizes the optimal wage-tenure policy of the firm:

\[ \left( -\frac{u''(w)}{u'(w)} \right) \frac{dw}{dt} = \lambda F'(W)Ju'(w) \quad (1.33) \]

Proof. See Burdett and Coles[2002].

The left side of equation (1.33), the product of the risk aversion coefficient and the difference between tomorrow’s and today’s wage, is the utility
cost to the worker of a small transfer of wage income from today to tomorrow. An increase in tomorrow’s wage increases the worker’s continuation value $W$ tomorrow by an amount $u'(w)$ and decreases the quit probability, $q(w) = \lambda [1 - F(W)]$, by $q'(w) = -\lambda F'(W)$. Hence, the right side is the saving in turnover cost to the employer expressed in utility terms attributable to small transfer of wage income from today to tomorrow. Note that equation (1.33) requires a rising wage-tenure profile, $dw/dt > 0$, because $F'(W) > 0, \forall W \in [W, \overline{W})$. In other words, firms respond to quit turnover of the employee by backloading wages in any optimal wage-tenure profile. Consequently, the value of employment also increases with tenure, i.e., $dW/dt > 0, \forall W < \overline{W}$.

Note, however, that the above inferences are based on the assumption that workers are risk averse, i.e., $u' > 0$ and $u'' < 0$. If we suppose instead that workers are risk neutral as in Stevens[2002], as which requires $u'' = 0$, we must have $F'(W) = 0, \forall W$ from equation (1.33). In other words, the equilibrium distribution of contract offers is degenerate in this limiting case, as shown by Stevens[2002]. This degenerate equilibrium implies all firms offer the same wage-tenure contract and there is no quit turnover. Even so, the limiting wage tenure contract is a smooth, strictly increasing function of tenure($dw/dt > 0$). Indeed, as all firms offer $W = U$, workers obtain no surplus through finding employment whereas employers extract all match rents, the Diamond’s[1971] monopsony solution is obtained in this limit.

In contrast, it can be shown analytically that as the coefficient of risk aversion becomes infinite, the market equilibrium converges to the Burdett-Mortensen[1998] equilibrium\(^9\). In this limit, the insurance effect dominates so that wage tenure effects become arbitrarily small. Infinitely risk averse workers imply a flat wage-tenure contract is optimal($dw/dt = 0$). All individual worker wage growth arises through quitting to better paid employment. Given that the offer arrival rate $\lambda$ is the same for employed and unemployed workers, equilibrium implies $w = b$.

\(^9\)When we consider the special case where workers have a constant relative risk averse utility function, we will find $\lim_{\rho \to \infty} (1 - \frac{\rho}{w}) = \lim_{\rho \to \infty} \rho/\rho = \infty$. So we must have $\lim_{\rho \to \infty} dw/dt = 0$ because $F'(W) < \infty, \forall W$.\(^9\)
Now return to the basic assumption workers are risk averse. The optimal wage-tenure contract has the following characteristics. First, as $dW/dt > 0$ if $W \in [\underline{W}, \overline{W})$ but $dW/dt = 0$, the expected discounted values of a contract and a match, $W$ and $J$, satisfy the boundary conditions:

$$u(\overline{W}) = (r + \delta)\overline{W} - \delta U$$
$$J(\overline{W}) = \frac{p - \overline{w}}{r + \delta}$$

Note that we must have $\overline{w} < p$, i.e., firms extract strictly positive profit in equilibrium. Next, because all firms and workers are respectively identical ex ante, the iso-profit condition requires all optimal wage-tenure contracts yield the same maximum expected profit. Formally,

$$\pi(\overline{W}) = \left\{ \begin{array}{l l}
\pi(\overline{W}) & if \ W_0 \in [\underline{W}, \overline{W}]\\
\leq \pi(\overline{W}) & otherwise
\end{array} \right.$$  

where $\pi(\overline{W}) = J(\overline{W}) = \frac{p - \overline{w}}{r + \delta}$. Third, because unemployed workers accept all contracts above the value of unemployment, the support of optimal wage-tenure contract is bounded in the lower limit at:

$$\underline{W} = U$$

Last, given any initial value of contract offered $W_0$ randomly drawn from the support $[\underline{W}, \overline{W}]$, a labor market equilibrium is a vector of function $\{w(t), W(t), J(t), F(W(t)), G(W(t))\}$ that converges to its stationary solution asymptotically. That is,

$$\lim_{t \to \infty} \{w(t), W(t), J(t), F(W(t)), G(W(t))\} = \{\overline{w}, \overline{W}, \overline{J}, 1, 1\}$$

**Equilibrium solution: baseline salary scale** The market equilibrium in Burdett-Coles[2004] is solved by a baseline salary scale. Simply speaking, the baseline salary scale is the equilibrium wage-tenure profile of a firm offering the lowest starting wage for new hires in the market. Indeed, any other firm’s wage-tenure profile can be described by this baseline salary scale with a different starting point. For example, suppose a firm offers a starting wage that is the same as the baseline salary at (say) 7 months tenure. An optimal contract implies employees at this firm are paid a wage after 3 months equal
1.6. INTRA-FIRM WAGE DISPERSION

to the wage paid at 10 months according to the baseline salary scale. And so on. This, of course, leads to a strong testable restriction on possible wage-tenure profiles offered by firms.

Formally, the baseline salary scale is a solution to the vector of function \( \{ w(t), W(t), J(t), F(W(t)), G(W(t)) \} \) given that \( W_0 = \bar{W} \). Let denote the market equilibrium solution based on the baseline salary scale as \( \{ w^*(t), W^*(t), J^*(t), F^*(W(t)), G^*(W(t)) \} \). Note that \( w^*(t) \) and \( W^*(t) \) are both continuous and increasing functions of tenure \( t \) and converge to \( \bar{w} \) and \( \bar{W} \) respectively. Hence, given any starting value of contract offered \( W_0 \in [\underline{W}, \bar{W}] \), a tenure point \( t_0 \geq 0 \) exists where \( W_0 = W^*(t_0) \). Furthermore, as \( W_0 = W^*(t_0) \) implies \( dW_0/dt = dW^*(t_0)/dt \) from equation (1.31), the optimal wage contract \( W(t) \) given starting value \( W_0 \) corresponds to the baseline contract for tenures \( t_0 + t \), i.e., \( W(t) = W^*(t_0 + t) \). In other words, any optimal contract given initial value \( W_0 \) is the remainder of the baseline contract starting from tenure \( t_0 \).

Given this insight, let’s reconsider market equilibrium. Let \( F^*(t) \) denote the distribution of starting points on the baseline salary scale. Since \( W_0 = W^*(t_0) \), one can define the probability that an offer starts at \( t \) or less on the baseline salary scale as \( F^*(t) = F(W^*(t)) \). Differentiating \( F^*(t) \) with respect to \( t \) yields:

\[
F'(W^*(t)) = \frac{dF^*(t)}{dt} \frac{dW^*(t)}{dt}
\]

Note that the iso-profit condition requires:

\[
\pi(W^*(t)) = h(W^*(t))J^*(t) = \pi(\bar{W}) = \frac{p - \bar{w}}{r + \delta}
\]

By differentiating this equation with respect to \( t \), one obtains:

\[
h'(W^*(t)) \frac{dW^*(t)}{dt} J^*(t) + h(W^*(t)) \frac{dJ^*(t)}{dt} = 0
\]

In other words, the positive effect of a more generous contract offer, a higher starting point \( t \), on the acceptance probability is just off set by a corresponding decline in the initial value of the match. After substituting appropriately \( \frac{dW^*(t)}{dt} \) and \( \frac{dJ^*(t)}{dt} \) out of the above equation and rearranging the terms, one
obtains:

\[
\frac{dF^s(t)}{dt} = \frac{\delta(r + \delta)}{\lambda} \left( \frac{p - w^s(t)}{p - \bar{w}} - \frac{(\delta + \lambda[1 - F^s(t)])(r + \delta + \lambda[1 - F^s(t)])}{\delta(r + \delta)} \right)
\]

(1.34)

The distribution of starting points on the baseline salary scale, \(F^s(t)\), is fully characterized by the above differential equation associated with the boundary condition \(F^s(0) = 0\).

To summarize the market equilibrium, we have the following proposition:

**Proposition 1.** A market equilibrium is a vector of functions mapped into the baseline wage scale, \(\{w^*, W^*, J^*, F^*, G^*\}\), which converges to its stationary solution \(\{\bar{w}, \bar{W}, \bar{J}, 1, 1\}\) asymptotically. There exists only an unique labor market equilibrium.

Proof. Let’s construct the ODE(ordinary differential equations) system:

\[
\frac{dF^s(t)}{dt} = \frac{\delta(r + \delta)}{\lambda} \left( \frac{p - w^s(t)}{p - \bar{w}} - \frac{(\delta + \lambda[1 - F^s(t)])(r + \delta + \lambda[1 - F^s(t)])}{\delta(r + \delta)} \right)
\]

\[
\frac{dw^s(t)}{dt} = \lambda \left( \frac{|u'(w^s)|}{-u''(w^s)} \right) \left( \frac{J(\delta + \lambda(1 - F^s))}{\delta} \right) \left( \frac{dF^s(t)}{dt} \right)
\]

\[
\frac{dW^s(t)}{dt} = (r + \delta)W^s(t) - u(w^s) - \delta U - \lambda S^s(t)
\]

\[
\frac{dS^s(t)}{dt} = -(1 - F^s)\frac{dW^s(t)}{dt}
\]

where \(S^s(t)\) is defined as:

\[
S^s(t) = \int_{W^s(t)}^{\bar{W}} [x - W^s(t)]dF(x)
\]

and \(\bar{J}, U\) are the solutions of:

\[
\bar{J} = \frac{p - w^s(\infty)}{r + \delta}
\]

\[
rU = rW^s(0) = u(b) + \lambda S^s(0)
\]

This ODE system, defined on the support \(t \in [0, \infty)\), is restricted by the
following boundary conditions:

\[ F^s(0) = 0 \]
\[ u(w^s(\infty)) = (r + \delta)W^s(\infty) - \delta U \]
\[ W^s(0) = U \]
\[ S^s(\infty) = 0 \]

In equilibrium, there must exist an unique solution to this ODE system. □

The next proposition characterizes the shape of equilibrium wage distribution:

**Proposition 2.** The densities of contracts and starting wage offers, \( F'(W) \) and \( F'(w) \), have both a right tail that tends to zero at their upper support. Formally, \( \lim_{W \to \infty} F'(W) = 0 \) and \( \lim_{w \to \infty} F'(w) = 0 \).

**Proof.** Note that all optimal contracts converge to its stationary limit only asymptotically, i.e.,

\[ \lim_{t \to \infty} \frac{dW^s(t)}{dt} = 0 \]

From the proposition 1, we have

\[ \frac{dw^s}{dt} = \left( \frac{[u'(w^s)]^2}{-u''(w^s)} \right) \frac{\lambda \bar{J}(\delta + \lambda (1 - F^s))}{\delta} \frac{dF^s}{dW^s} \]

Then

\[ \lim_{t \to \infty} \frac{dF^s(t)}{dw^s(t)} = \lim_{t \to \infty} \frac{dF^s(t)}{dt} \frac{dt}{dw^s(t)} = \lim_{t \to \infty} \left\{ \left\{ \left[ \frac{[u'(w)]^2}{-u''(w)} \right] \frac{\lambda \bar{J}(\delta + \lambda)}{\delta} \right\}^{-1} \frac{dW^s(t)}{dt} \right\} = 0 \]

Similarly, we will also have

\[ \lim_{t \to \infty} \frac{dF^s(t)}{dW^s(t)} = 0 \]

□
1.6.2.3 Discussion

Following Burdett-Coles[2004], we consider the case where workers have a constant relative risk averse (CRRA) utility function:

\[
    u(x) = \frac{x^{1-\rho}}{1 - \rho}
\]

where \( x = w, b \) and \( \rho > 0 \). Figure 1.9 plots the equilibrium distributions of wage earnings and staring wage offers, predicted by the Burdett-Coles[2004] model.

Why is there intra-firm wage distribution and wage growth? In Postel-Vinay and Robin[2002], wages rise with tenure because employers pay each worker a lower starting wage (his reservation wage) initially and then raise the wage to match outside offers. Although employers do not match outside offers in Stevens[2002] and Burdett-Coles[2004], they do find it optimal to offer back-loaded wage-tenure schedules (two-tier contract in the former and smoothly increasing wage-tenure profile in the latter) to control inefficient worker turnover. As a consequence of the wage-tenure relation, wage dispersion arises both within and across firms.

All the three papers share a common characteristic. Namely, firms seek to control inefficient quit turnover of their labor force. However, massive flows of worker mobilities are typically observed at the aggregate level. This evidence means that firms, indeed, cannot rule out inefficient separation even though they have monopsony market power\(^{10}\). Therefore, investigating how and to what extent firms respond to the moral hazard problem when employees receive better outside offers, as well as the resulting incidences to social welfare, is an empirical subject of great interest.

Whether or not seniority has a large effect on wage growth has been the subject of continuing controversy. Recent studies show that the returns to seniority are rather modest\(^{11}\). Altonji-Shakotko[1987] estimate that on-the-job wage growth is 6.6% per decade. Altonji-Williams[1997,2004] after surveying alternative estimates of wage growth state that the return to ten years of

\(^{10}\) Clearly, labor protection legislation is not the unique reason.

\(^{11}\) see Altonji and Williams[1997,2004], Bowlus, Kiefer and Neumann[2001], Bowlus-Neumann[2004] for some recent surveys.
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Figure 1.9: Equilibrium wage distributions (Burdett-Coles[2004])

Illustrative calibration: $\lambda = 0.3$, $\delta = 0.1$, $w = 1$, $b = 0.8$, $\rho = 4$, $p = 5.5$

tenure of about 0.11, i.e., 1.1% per year. Using U.S. census data (NLSY 1979-1994), Bowlus-Neumann[2004] examine on-the-job wage growth and find that young males see wage growth rates between 5-6% per annum over the first 10 years of labor market experience. Importantly, the majority of this growth does not stem from on-the-job wage growth. On average, on-the-job wage growth ranges from negligible to at most 2.5% per annum. In contrast, a single job change increases wages between 9-13% on average. All of these studies indicate that on-the-job wage growth can account for only a small share of total wage growth. Putting it differently, much more attention should be paid to job-to-job transitions when studying wage growth.

1.7 Conclusion

In this chapter, we survey the recent developments of equilibrium search theory and focus on a fundamental question of labor economics: why are workers paid differently? We first present the empirical studies that investigate the determinants of wage differentials and wage variation. We then explain wage dispersion along four lines: pure wage dispersion, wage dispersion linked with exogenous or endogenous heterogeneity, and intra-firm wage dispersion and wage growth.

The studies of Abowd-Kramarz[2000a,b], Postel-Vinay and Robin[2002]
and Van den Berg-Van Vuuren[2003] all indicate that wage differentials are fundamentally driven by five factors: individual heterogeneity (measured and unmeasured), firm heterogeneity (measured and unmeasured), and market friction. Among the five factors, observable firm heterogeneity and unobservable firm heterogeneity are both important determinants of wage variation. The share of the cross-sectional wage variance that is explained by person effects varies across skill groups. In contrast, pure friction effect typically explains no more than 10% of wage variation in the data, the residual variance is attributed to an interaction effect between market friction and firm/worker heterogeneity. The theoretical investigations confirm that job search models that allow for a continuous productivity dispersion will provide better coincidence with the wage data. We then conclude that observed wage dispersion principally reflects differences in employer characteristics (measured and unmeasured) in interaction with market frictions.

It should be stressed that search friction is the first determinant of wage dispersion. In a hypothetical frictionless market, all workers will instantaneously find the highest wage offer and all firms will instantaneously fill a vacancy, there will be neither unemployment nor vacant jobs in economics. Productivity variation across firms within a market could by itself not generate any wage dispersion, in the sense that wage dispersion may equal zero if frictions are infinitely large or absent. On the other side, we should also remark that wage dispersion will vanish if all firms are identical (in both measured and unmeasured characteristics), i.e., if all firms share the same technology of production and if there are no differences in wage policy across firms. Therefore, it is rather the interaction between firm heterogeneity and market frictions that provides a good fit to within-market wage distributions.
Appendix A: Statistical model in Abowd and Kramarz[2000a,b]

A.1 Basic wage equation

The dependent variable is compensation, $y_{it}$, observed for individual $i$ at date $t$ and measured as a deviation from its grand mean $\mu_y$. This variable is expressed as a function of individual heterogeneity, firm heterogeneity and measured time-varying characteristics:

\[
(y_{it} - \mu_y) = \theta_i + \psi_{jit} + (x_{it} - \mu_x)\beta + \varepsilon_{it}
\]  

(A-1)

The first component $\theta_i$ is the time-invariant individual effect. The second component, $\psi_{jit}$, is the firm effect for the firm $j$ at which worker $i$ is employed at date $t$. The vector $x_{it}$ contains all time-varying observable characteristics of individual $i$ at date $t$, also measured as a deviation from the grand mean $\mu_x$, $\beta$ is a vector of worker characteristic coefficients. The fourth component is the statistical residual, $\varepsilon_{it}$, orthogonal to all other effects in the model.

The person effect, $\theta_i$, combines the effects of observable time-invariant personal characteristics, $u_i$, and unobserved personal heterogeneity $\alpha_i$:

\[
\theta_i = \alpha_i + u_i\eta
\]

where $\eta$ is a vector of effects associated with the time-invariant personal characteristics.

The firm effect, $\psi_{jit}$, combines the effects of unobservable firm heterogeneity, $\phi_j$, time-variant observable firm characteristics, $q_{jt}$, and time-varying employer-employee match characteristics, $s_{jit}$:

\[
\psi_{jit} = \phi_j + q_{jt}\rho + s_{jit}\gamma_j
\]

where $\rho$ is a vector of effects associated with the time-variant firm characteristics, $\gamma_j$ is a vector of parameters associated with the match characteristics.
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A.2 True industry and size effects

A true industry (or size) effect, denoted by $\kappa_k$, is an average of the firm effect parameters within the industry (or size class) $k$. Formally,

$$\kappa_k = \frac{1}{N_k} \sum_{i=1}^{N} \sum_{t=1}^{T} \left[ \frac{1(K(j) = k)}{N_k} \psi_{jit} \right]$$

where

$$N_k = \sum_{j=1}^{J} 1(K(j) = k) N_j$$

The function $K(j)$ denotes the industry (or size) classification of firm $j$, function $J(i, t)$ denotes the firm at which an individual $i$ is employed at date $t$, function $1(\cdot)$ takes the value 1 if true and 0 otherwise.

If we insert the true industry (or size) effect into the mean wage equation (A-1), then the equation becomes:

$$ (y_{it} - \mu_y) = \theta_i + (\psi_{jit} - \kappa_k) + \kappa_k + (x_{it} - \mu_x)\beta + \varepsilon_{it} \quad (A-2) $$

The firm effect, $\psi_{jit}$, is then decomposed into two orthogonal components: the industry (or size) effects, $\kappa_k$, and what is left of the firm effects after removing the industry (or size) effect, $\psi_{jit} - \kappa_k$.

A.3 Raw industry and firm effects

However, most studies cannot identify the firm effect because employer identities are not available in the data set used to estimate the model. For such data Abowd-Kramarz[2000a,b] propose the following form of wage equation:

$$ (y_{it} - \mu_y) = \kappa_{k}^{**} + (x_{it} - \mu_x)\beta + \varepsilon_{it} \quad (A-3) $$

Abowd-Kramarz[2000a,b] refer $\kappa_{k}^{**}$ to the “raw” industry (or size) effect. The raw industry (or size) effect is a weighted average of the person and firm effects within the industry (or size class) of interest. Abowd-Kramarz[2000a,b] show that the respective contribution of person and firm effects could be exactly estimated in this case.
A.4 Decomposition of raw industry and firm effects

We rewrite the true industry and size effects model (A-2) into matrix form as:

$$Y = D\Theta + (F\Psi - FA\kappa) + FA\kappa + X\beta + \varepsilon$$

where the matrix $D$ is the design matrix for the worker effect, the matrix $F$ is the design matrix for the employer effect, the matrix $A$ is the classification matrix that maps firms into industries. Thus, the matrix $FA$ is the design matrix for the true industry effect.

The true industry effect $\kappa$ can be expressed as:

$$\kappa = (A'F'M_XF_A)^{-1}A'F'M_X(FA\Psi + D\Theta)$$ (A-4)

which is just the appropriately weighted average of the firm effects within the industry.

Abowd-Kramarz-Margolis[1999] show that the raw industry effect $\kappa^{**}$ is:

$$\kappa^{**} = \kappa + (A'F'M_XF_A)^{-1}A'F'M_X(M_{FA}F\Psi + D\Theta)$$ (A-5)

where the matrix $M$ is the residual matrix (column null space) after projection into the column space of the matrix in the subscript, $X$:

$$M_X = I - X(X'X)^{-1}X'$$

The equation (A-5) means that the vector $\kappa^{**}$ of industry effects can be expressed as the true industry effect $\kappa$ plus a bias that depends upon both the person and firm effects. Abowd-Kramarz-Margolis[1999] further decompose the raw inter-industry (or firm size) wage effect into the sum of the industry-average person effect and the industry-average firm effect, both conditional on $X$:

$$\kappa^{**} = (A'F'M_XF_A)^{-1}A'F'M_XD\Theta + (A'F'M_XF_A)^{-1}A'F'M_XF\Psi$$ (A-6)

by equation (A-4). Thus, the vector $\kappa^{**}$ of raw industry effects can be expressed as a matrix weighted average of the person effects $\Theta$ and the firm
effects $\Psi$. The decomposition of the raw inter-industry (or firm-size) wage differentials is:

\[
\text{Part due to individual effects} = \frac{(A'F'M_XFA)^{-1}A'F'M_XD\Theta}{\kappa^{**}} (A-7)
\]

\[
\text{Part due to firm effects} = \frac{(A'F'M_XFA)^{-1}A'F'M_XF\Psi}{\kappa^{**}} (A-8)
\]

Equation (A-6) is exact if the values of $\Theta$ and $\Psi$ are known. Abowd-Finer-Kramarz[1999] and Abowd-Kramarz-Margolis[1999] show that if statistical approximations are used, then equation (A-6) provides a consistent estimate of the decomposition for the sample. In Abowd-Kramarz[2000a,b], the authors use exact least squares estimates to estimate the two sets of effects (estimated simultaneously). The authors find that the equation (A-6) is essentially exact for their sample because the two components are estimated with great precision for each industry.
Chapter 2

Relation between wage earnings and job-to-job transitions: empirical evidences

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2.1 Introduction

Nowadays, equilibrium search-matching theory has renewed the way how economists understand the labor market. As discussed in the first chapter, search friction, imperfect information and non-cooperative competition among firms constitute the theoretical core of this theory. Specially, these concepts are crucial to understand why unemployed workers could contact only a subset of total wage offers, why firms take time to fill a vacancy, and why identical workers (in both observable and unobservable characteristics) in identical firms end up experiencing different wage trajectories. As a result of market friction, wage dispersion is a robust equilibrium outcome provided that workers search on-the-job.

From the end of the 1990s, many empirical studies have tested different versions of equilibrium search-matching model, in particular using French linked employer-employee data. Bontemps-Robin-Van den Berg[2000] first provide an structural estimation of the Burdett-Mortensen[1998] model with continuous distribution of employer types. Using data from French Labor Survey, Bontemps-Robin-Van den Berg[2000] establish that the Burdett-Mortensen[1998] model provides a good fit to the wage data once productivity dispersion is included into the model. In contrast, the model without productivity dispersion cannot explain more than 10% of total wage variation in the data. To put this differently, the interaction effect of on-the-job search and productivity dispersion is a more important determinant of wage dispersion than the marginal effect of on-the-job search. Postel-Vinay and Robin[2002] extent the Burdett-Mortensen[1998] model to allow for heterogeneity in both firm productivity and worker capacity. The authors then estimate their model and find that the model fits the wage data quite well. Particularly, Postel-Vinay and Robin[2002] state that heterogeneity in firm productivity is the most important determinant of wage variation. Cahuc, Postel-Vinay and Robin[2006] modify the Postel-Vinay and Robin[2002] model by introducing

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wage bargaining as a wage determination rule. Their main finding is that between-firm competition plays a prominent role in wage determination in France over the period 1993-2000. The bargaining power of workers turns out to be very low—typically between 0 and a third—in all industries. However, they definitely find that skilled workers have significant bargaining power. To sum up, all of these studies focus on the empirical performance of the equilibrium search model in explaining wage dispersion.

Let's inspect this theory along another line. Equilibrium search-matching theory typically views labor markets as job ladders\(^2\). Workers search for good matches while unemployed and while employed. Employment with a particular firm ends either when the job is destroyed or when a worker finds a better opportunity. This simple structure yields several strong predictions. Incomes increase as workers move from lower paying to higher paying jobs, occasionally interrupted by spells of unemployment as some jobs are destroyed and the worker has to start over again on the lower rungs of the ladder. Therefore, the probability of job-to-job transition always decreases with the wage of the worker. Of course, this job ladder view of the labor market is necessarily too simplistic in some dimensions. Detecting what those dimensions are and to what extent, and understanding why the job ladder paradigm fails are all issues of great importance.

The first work that systematically inspects the empirical pertinence of the Burdett-Mortensen[1998] model along this direction is seen in Bowlus-Neumann[2004]. Using U.S. census data(NLSY 1979-1994), the authors find that, on average, 40% of all job-to-job transitions result in lower wages and almost 30% result in a wage decline of more than 5%. Surprisingly, the frequency of job-to-job transitions is indeed increasing with wages earnings, and the frequency of job-to-job transitions that result in a negative wage change also increases with wage earnings. These evidences clearly contradict the model prediction that job-to-job transition always result in a wage increase.

\(^2\)Examples include Burdett-Mortensen[1998], Mortensen[2002a], Rosholm-Svarer[2004], Robin-Roux[2002], Mortensen[2002b], Cahuc, Postel-Vinay and Robin[2006], Burdett-Coles[2004]. The sole exception is Postel-Vinay and Robin[2002] where job-to-job mobility associated with wage decline is allowed and possible. However, the pertinency of this result in the data is not discussed in their paper.
The work of Bowlus-Neumann[2004] reveals a dimension until now largely neglected: the relation between wage earnings and job-to-job transitions. Do the results of Bowlus-Neumann[2004] apply also to the French labor market? If so, to what extent? What are the distinct characteristics of the job-to-job transition flows in the French labor market? In this chapter, we examine at length all these issues using French panel data (French Labor Survey, or Enquête Emploi).

We decompose the total flows of job-to-job transition into flows of promotion and flows of external mobility. Formally, we consider three types of job-to-job mobility: external mobility (without promotion), external promotion and internal promotion. By promotion, we mean a raise of occupational category of the employee in sense of Social Professional Category (SPC). By external mobility, we mean a job-to-job mobility via a change of firm (made with an episode of unemployment of one month or less). Importantly, an internal promotion should also be considered as a job-to-job transition because it concerns a change of job (see Lazear-Oyer[2004]).

We then classify the individuals by their occupational category (Socio-Professional Category, or SPC). We find that the frequency of external mobilities is unambiguously decreasing with wage earnings for any SPC in consideration. This empirical evidence confirms the prediction of equilibrium search theory that external mobility is a decreasing function of wage. However at the aggregate level, i.e., if we don’t classify the individuals by SPC, the relation between frequency of external mobilities and wage is rather ambiguous. We then suspect that the increasing relation between frequency of external mobilities and wage in Bowlus-Neumann[2004] is due to the composition effect.

We also find that the frequency of promotions is clearly increasing with wage earnings for any SPC in consideration. In the 1990s, around two thirds of job-to-job transitions are in nature promotion in sense of a raise of occupational category (SPC). Further, around 90% of promotions are made within firms, whereas only 10% are made via a change of firm. Therefore, internal promotions account for around 60% of total job-to-job transitions. Nevertheless, most equilibrium search models don’t allow to deal with this particular
type of professional mobility.

We distinguish, among the flows of job-to-job transition, the job changes associated with wage increase from those with wage cut. We find that, in the decade 1990-1999, around 40% to 55% of job-to-job transitions result in lower wages in new jobs. This number is similar to the finding of Bowlus-Neumann[2004](40%) using U.S. data. We also find that the actual relation between the frequency of job-to-job transition and wage earnings is typically U-shaped. This U-shaped relation refers to two particular types of job-to-job mobility that most equilibrium search models fail to capture, i.e., mobilities that result in either a wage decrease or a raise of occupational category. However, once these two types of mobility are discarded from the total flows, the frequency of the residual mobilities—the external mobilities that result in a wage increase but not a raise of occupational category—tend to decrease with wage earnings.

To present all these statistical results orderly, this chapter is organized as follows. Section 2.2 presents the data, classifies and describes the types of mobility to be studied. The preliminary results found in section 2.2 lead to a further research in section 2.3 about the relation between wage earnings and frequency of job-to-job mobility. In section 2.4, we estimate the Burdett-Mortensen[1998] model while allowing for a continuous productivity distribution, and compare the wage distribution and the labor mobilities predicted by the model with those observed in the data. We turn to a simulated method of moments (SMM) to estimate the structural model. Finally, the section 2.5 concludes.

2.2 Evidences of wage distribution and job-to-job transition

2.2.1 Data

Our data source is the French Labor Survey(FLS, or Enquête Emploi). FLS is the most important source that permits to measure employment, unemployment and non-activity in France. Besides, FLS collects extensive infor-
mation on the labor market behavior of the individual respondents in the year preceding the moment of the interview. Generally speaking, the following themes are covered in FLS: identifier of survey, individual’s identifier, socio-professional category, sector of activity, status (employment or unemployment), number of hours worked, monthly wage and earnings, secondary activity, education, residence, situation of family, geographical origin and mobility, anterior occupation, and social origin.

For our empirical study we use data issued from the series 1990-1999. In this decade, the survey is carried out in January every year. It interviewed 150,000 peoples who are 15 years old or plus, residing in around 70,000 French families. The sampling rate is then around 1/400. The sample is partially renewed every year (a third is dropped) so that an individual is interviewed in three consecutive years.

We restrict our sample to male French workers aged between 20 and 59 (before retirement at 60) who have a full-time jobs in the private sectors. This selection is motivated to control the effects of labor supply on wage and job trajectories. In particular, women are dropped from our sample as their wage and job trajectories deviate, at least in certain periods, substantially from the trajectories described by the basic equilibrium search theory.

All recorded monthly wages, net of tax and controlled by inflation and GDP growth, are transformed to constant Euro based on the level 1999.

2.2.2 Classification of individuals by SPC

Individuals are classified by their occupational category, or Socio-Professional Category (SPC), so as to characterize the wage distribution and professional mobility on the basis of the professional qualification of individuals. This classification allows to avoid the problems of composition effect in the definition of the aggregate indicator of professional mobility.

In total, seven principal categories are recorded in the data, they are: agricultural, self-employed, managers, intermediate professions, sales and service employees, skilled manual workers and unskilled manual workers. The SPC “managers” include all the executives, engineers, administrative and sales supervisors, and all high-ranking managers. Correspondingly, the SPC “in-
Figure 2.1: Distribution of individuals by SPC

Distribution by SPC in 1990

Distribution by SPC in 1998
2.2. FLOWS OF JOB-TO-JOB TRANSITION

Intermediate professions” consists of the technical supervisors and technicians, and all middle-ranking managers in administration and sales. In this study, as our attention focuses on male French workers in the private sectors, individuals who are self-employed or in the occupation “agricultural” are first dropped. Furthermore, it’s observed from the data that over the 1990s, women represent on average more than 80% of individuals in the SPC “employees”. In view of the number of observations, we could identify merely few frequencies of job-to-job transition occurred among those male “employees”. For these reasons we dropped also the SPC “sales and service employees” from our sample. So finally, all individuals belong to one of the following four SPC are retained in the sample: (1) managers (denoted by MAN), (2) intermediate professions (INT), (3) skilled manual workers (SW), (4) unskilled manual workers (USW).

The figure 2.1 plots the distribution of individuals in these four SPC for the year 1990 and 1998. This allows to show that the composition of labor force remains stable in this decade.

2.2.3 Wage distribution

The wage data is processed as follows. In the first step, we construct a separate database for every yearly survey of this decade. Wages are calculated based on recorded monthly wage earnings, net of tax and controlled by inflation and GDP growth, then transformed to constant Euro based on the level 1999. The mandatory minimum wage rates of every year over this decade are known from institutional sources. The raw data contain some full-time wages below the mandatory minimum wage. To deal with this, we replace all full-time wages below the mandatory minimum wage by missing values. In the second step, within each one of the 10 databases we keep only the observations presented at two successive dates of interview, \( t \) and \( t + 1 \), and drop all the other observations. In the third step, we combine the 10 separate databases into an integrated database by appending them, and equally classify individuals by SPC. We then normalize to unity the lowest wage observed in each SPC (see figure 2.2). Finally, we also trim the high-wage data by discarding the 1% largest wages of the sample per SPC.
Figure 2.2: Actual wage densities

Figure 2.3: Actual distribution of seniority
2.2. FLOWS OF JOB-TO-JOB TRANSITION

The figure 2.2 plots kernel estimates of the actual wage earnings distribution, averaged over the decade 1990-1999, for each of four SPC. Obviously, the shape of wage density is systematically uni-modal and of log-normal type.

2.2.4 Classification of professional mobility

We divide all job transitions into two categories: job-to-job transitions and job transitions via unemployment. This has been a standard way to identify transitions due to job destruction, i.e., those via unemployment, and transition due to on-the-job search. In this study, we aim to compare the wage and transition trajectories of the worker predicted by the equilibrium search model with the actual data. In line with this objective, we will focus on the flows of job-to-job transitions in the following text, whereas the flows of job transitions via unemployment are neglected.

Formally, the probability of job-to-job transition is defined as the yearly frequency of job transitions that are made within one month or less among the individuals presented at two successive dates of interview, $t$ and $t + 1$. Furthermore, we subdivide total job-to-job transitions into three categories:

- external mobility (change of firm) without promotion
- external mobility with promotion
- internal mobility (within the same firm) with promotion

Importantly, an internal promotion should also be considered as a job-to-job transition because it concerns a change of job (see Lazear-Oyer[2004]).

Of course, the sum of these three flows of mobility equals the total flows of job-to-job transition, the sum of the first and the second is the flows of external mobility, and the sum of the second and the third is the flows of promotion.

As the FLS databases provide no information about the employer’s identifier, the external mobility is identified via the seniority of the individual in firm. We consider an external mobility occurred if an employee, given that he is employed at $t$ and $t + 1$ and did not declare to experience a period of
unemployment more than 1 month between two successive dates of interview, reports at date $t + 1$ a seniority in firm inferior to one year\(^3\).

The figure 2.3 plots the histogram and kernel estimates of the actual distribution of seniority, averaged over the decade 1990-1999, for each of four SPC. This figure indeed reports the upper limit of the yearly frequency of external mobilities. Note that, in general, around 6% of workers report a seniority in firm less than one year for each one of four SPC in consideration.

Correspondingly, a promotion refers to a raise of occupational category of the individual, between years $t$ and $t + 1$. Precisely, three types of promotion are considered: (1) an unskilled blue collar becomes skilled blue collar (USW→SW), (2) a skilled blue collar accedes to a job classified in the SPC “intermediate professions” (SW→INT), (3) a middle-ranking manager becomes supervisor at $t + 1$ (INT→MAN). Other types of change of SPC are marginal in the sample, these including demotion (e.g. SW↓→USW) or upward promotion spanning two SPC (e.g. USW→SW→INT) between year $t$ and $t + 1$\(^4\).

All job-to-job transitions examined in this study refer to change of job. Further, we distinguish, among the flows of job-to-job transition, the job changes associated with wage increase from those with wage cut. Note that wages are calculated based on recorded monthly wage earnings, net of tax and controlled by inflation and GDP growth, then transformed to constant euro based on the level 1999. A wage increase (or cut) is then defined as a positive (or negative) wage change of at least one euro between year $t$ and $t + 1$.

The table 2.1 reports, by SPC and by mobility type, the summary statistics for all types of job-to-job transitions identified in the sample covering years from 1990 to 1999. In total, there are around 87,000 observations retained in the sample. Among these observations, more than 10,000 job-to-job transitions are accounted during the decade 1990-1999. In other words, the average yearly job-to-job transition rate is 12%.

\(^3\)We neglect the possibility of multiple job-to-job transitions within a year, as it is revealed from the data that such transitions among the full-time workers are extremely scare and marginal.

\(^4\)For any given individual, we could identify from the data at most one time the occurrence of promotion (or demotion) from one year to another.
All job-to-job transitions reported in Table 2.1 are defined as those made with an episode of unemployment of one month or less. For sake of comparison, we also report in Table 2.2 the summary statistics for job-to-job transitions that are made without an episode of unemployment. A comparison of these two tables reveals that most of job-to-job transitions are made without an episode of unemployment, those made with an episode of unemployment of one month are actually very scare.

Before further discussing, we restate that, in the following text, the probability of job-to-job transition is always defined as the yearly frequency of job transitions that are made with an episode of unemployment of one month or less among the individuals presented at two successive dates of interview, \( t \) and \( t + 1 \). Also, because only one occurrence of job-to-job transition could be identified from one year to another, our measure of job-to-job transitions represents the lower limit of the actual flows in France.
Table 2.1: Summary statistics for job-to-job transitions (transitions made with an episode of unemployment of one month or less)

<table>
<thead>
<tr>
<th>SPC</th>
<th>Observations</th>
<th>External mobilities without promotion</th>
<th>External promotions</th>
<th>Internal promotions</th>
<th>Total job-to-job transitions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Managers</td>
<td>12406</td>
<td>569</td>
<td>0</td>
<td>0</td>
<td>569</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>4.59</td>
<td>0</td>
<td>0</td>
<td>4.59</td>
</tr>
<tr>
<td>Intermediate professions</td>
<td>23299</td>
<td>874</td>
<td>106</td>
<td>1317</td>
<td>2297</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>3.75</td>
<td>0.45</td>
<td>5.65</td>
<td>9.86</td>
</tr>
<tr>
<td>Skilled manual workers</td>
<td>41475</td>
<td>1849</td>
<td>145</td>
<td>1763</td>
<td>3757</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>4.46</td>
<td>0.35</td>
<td>4.25</td>
<td>9.06</td>
</tr>
<tr>
<td>Unskilled manual workers</td>
<td>9637</td>
<td>282</td>
<td>236</td>
<td>3268</td>
<td>3786</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>2.93</td>
<td>2.45</td>
<td>33.91</td>
<td>39.29</td>
</tr>
<tr>
<td>Total</td>
<td>86817</td>
<td>3574</td>
<td>487</td>
<td>6348</td>
<td>10409</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>4.12</td>
<td>0.56</td>
<td>7.31</td>
<td>11.99</td>
</tr>
</tbody>
</table>
Table 2.2: Summary statistics for job-to-job transitions (transitions made without an episode of unemployment)

<table>
<thead>
<tr>
<th>SPC</th>
<th>Observations</th>
<th>External mobilities without promotion</th>
<th>External promotions</th>
<th>Internal promotions</th>
<th>Total job-to-job transitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Managers</td>
<td>12406</td>
<td>551</td>
<td>0</td>
<td>0</td>
<td>551</td>
</tr>
<tr>
<td>% Observations</td>
<td>100</td>
<td>4.44</td>
<td>0</td>
<td>0</td>
<td>4.44</td>
</tr>
<tr>
<td>Intermediate professions</td>
<td>23299</td>
<td>830</td>
<td>105</td>
<td>1317</td>
<td>2252</td>
</tr>
<tr>
<td>% Observations</td>
<td>100</td>
<td>3.56</td>
<td>0.45</td>
<td>5.65</td>
<td>9.67</td>
</tr>
<tr>
<td>Skilled manual workers</td>
<td>41475</td>
<td>1742</td>
<td>130</td>
<td>1723</td>
<td>3595</td>
</tr>
<tr>
<td>% Observations</td>
<td>100</td>
<td>4.20</td>
<td>0.31</td>
<td>4.15</td>
<td>8.67</td>
</tr>
<tr>
<td>Unskilled manual workers</td>
<td>9637</td>
<td>282</td>
<td>197</td>
<td>3075</td>
<td>3554</td>
</tr>
<tr>
<td>% Observations</td>
<td>100</td>
<td>2.93</td>
<td>2.04</td>
<td>31.91</td>
<td>36.88</td>
</tr>
<tr>
<td>Total</td>
<td>86817</td>
<td>3405</td>
<td>432</td>
<td>6115</td>
<td>9952</td>
</tr>
<tr>
<td>% Observations</td>
<td>100</td>
<td>3.92</td>
<td>0.50</td>
<td>7.04</td>
<td>11.46</td>
</tr>
</tbody>
</table>
2.2.5 Aggregate flows of job-to-job transition

Figure 2.4 reports the yearly job-to-job transition rate in the decade 1990-1999 (average for four SPC). During this decade, the yearly job-to-job transition rate fluctuates around 12% in France. Further, a substantial part of job-to-job transitions is associated with wage declines. In figure 2.4, the solid line represents the yearly transition rate, whereas the dashed line represents the frequency of job-to-job transition associated with wage increase. As a consequence, the gap between these two lines represents the frequency of job-to-job transitions that result in wage declines. This figure shows that in the decade 1990-1999, around 40% to 55% of job-to-job transitions result in lower wages in new jobs. This number is comparable to that in Biscourp-Dessy-Fourcade[2005](20%-30%) using French data.\footnote{Biscourp-Dessy-Fourcade[2005] study the evolution of wage in France at the end of the 1990s (1995-2000) using three different data sources: DADS, Enquête Emploi (FLS) and Enquêtes Revenus Fiscaux. They focus on individuals staying in the same firm between two dates of interview. Therefore, their study doesn’t consider wage change associated with external mobility. The authors find that the proportion of individuals confronting...}
2.2. FLOWS OF JOB-TO-JOB TRANSITION

Figure 2.5: Proportion of promotions in total job-to-job transitions

Figure 2.6: Yearly promotion rate
The flows of promotion constitute an important part in the total flows of job-to-job transition. The figure 2.5 indicates that, in the 1990s, around two thirds of job-to-job transitions are in nature promotion in sense of a raise of occupational category (SPC).

If we further subdivides the total flows of promotion into flows of internal promotion (within firms) and flows of external promotion (towards other firms), we find that internal promotion is the primary channel of promotion in France (see figure 2.6). Indeed, around 90% of promotions are made within firms, whereas only 10% are via a change of firm. From the same figure, we also observe that around 35-60% of promotions result in a wage decline, i.e., a decrease of real wage from one year to another.

Figure 2.7 reports the yearly average external mobility rate in the decade 1990-1999 (average for four SPC). During this decade, the yearly average external mobility rate is around 4.5% in France. This number clearly indicates that external mobility is a scare event in France, and confirms the finding of Jolivet, Postel-Vinay and Robin [2002] that the France is clearly a "low-
wage declines is of the same scope in all three data sources: all around 20%-30%. Note also that this number doesn’t include the fraction of zero wage change.
2.2. FLOWS OF JOB-TO-JOB TRANSITION

turnover" country similar to Belgium, Italy, Portugal and Spain\(^6\). Further, around 45% of external mobilities result in wage declines in new jobs, a number close to that in Bowlus-Neumann[2004](40%) using U.S. data. In other words, the average frequency of external mobilities that result in wage increases is around 2.5% in this decade.

A preliminary summary can already be established. It should be pointed out that the canonical equilibrium search models à la Burdett-Mortensen[1998] fail to capture a substantial part of total job-to-job transitions: this notably includes mobilities associated with wage decreases(around 40-55%). Furthermore, the question of promotions is not directly discussed by these models, whereas promotions account for around two thirds in the total job-to-job transitions.

Intuitively, these two types of mobility could be interlinked. Consider the case of an employee who actually occupies a job classified in SPC “intermediate profession”. It could be optimal for him to accept a “supervisor” job even though this job change results in a real wage decrease today, because he envisages the potential of within-firm wage growth in the future. Argument of this type is first proposed by Postel-Vinay and Robin[2002]. The authors explain negative wage growth via job-to-job changes by emphasizing intra-firm wage growth. Nevertheless, the empirical pertinency of this mechanism is not discussed in their paper.

2.3 Evidences of wage-transition relation

Most of job search models view labor markets as job ladders: unemployed workers search for job offer whereas employees search for wage increase. Because a higher wage will reduce the probability that an employee accepts job offers from other firms, the models predict a negative association between

\[^6\]Using European data from ECHP and U.S. data from PSID, Jolivet, Postel-Vinay and Robin[2002] find that the set of countries investigated in their study could be divided into a clearly “high job-to-job turnover” category which comprises Denmark and the U.K., a clearly “low-turnover” group with Belgium, France, Italy, Portugal and Spain, and finally a “middle-range” group, with Germany, the Netherlands, the U.S. and Ireland—the latter two countries being closest to the “high job-to-job turnover” category.
CHAPTER 2. EMPIRICAL EVIDENCES

Empirical studies that provide structural estimation of different versions of equilibrium search model, especially those of Bontemps, Robin and Van Den Berg[2002], Christiansen et al.[2005], Rosholm-Svarer[2004], Robin-Roux[2002], Postel-Vinay and Robin[2002] and Cahuc, Postel-Vinay and Robin[2006], mainly aim at examining their model's capacity in reproducing actual wage distribution. Nevertheless, the performance of these models in reproducing actual transition data is, to a great extent, a neglected aspect.

Contrarily to these work, we will focus on the empirical relation between wage earnings and frequency of job-to-job transitions (as well as transitions associated with wage cuts). This study covers all occupational categories in consideration and all types of job-to-job mobility.

2.3.1 Relation between wage earnings and frequency of external mobilities

In the last section, all indicators of job-to-job transition are defined on the aggregate level (average for four SPC). In this section, we de-aggregate these indicators by SPC. This way allows to avoid the composition effect. These de-aggregated indicators are calculated as the average over the 10 years period.

Figure 2.8 plots the frequency of external mobilities (with and without promotion) by wage decile for each one of four SPC in consideration. Definition of external mobility of this type is a standard way that we usually see in the literature (including in Bowlus-Neumann[2004]). This figure indicates that the frequency of external mobilities, as well as mobilities associated with wage increases, is unambiguously decreasing with wage earnings for any SPC in consideration. This empirical evidence confirms the prediction of equilibrium search theory that external mobility is a decreasing function of wage.

However at the aggregate level, i.e., if we don’t classify the individuals by SPC, the relation between frequency of external mobilities and wage decile is rather ambiguous (see figure 2.9). For wages greater than 6th decile, the
2.3. WAGE-TRANSITION RELATION

Figure 2.8: Frequency of external mobilities by wage decile

![Graphs showing frequency of external mobilities by wage decile for different professions: Manager, Intermediate Profession, Skilled Manual Worker, Unskilled Manual Worker.]

NB: 10=1st decile; 20=2nd decile; 100=10th decile.

Figure 2.9: Frequency of external mobilities by wage decile: at the aggregate level

![Graph showing frequency of external mobilities by wage decile for different wage deciles at the aggregate level.]

NB: 10=1st decile; 20=2nd decile; 100=10th decile.
frequency of external mobilities tend to increase with wage decile. Clearly, this result is due to a composition effect: in figure 2.9, the external mobility rates that correspond to the 9th and 10th deciles mainly reflect the average external mobility rates of the highest-paid employees of the SPC “manager” and “intermediate profession”, whereas the external mobility rates that correspond to the 1st and 2nd deciles mainly reflect the average external mobility rates of the lowest-paid employees of the SPC “unskilled manual worker” and “skilled manual worker”. Also it should be pointed out that, at the aggregate level, the shape of the relation between external mobility and wage decile depends on the weight of each SPC (the number of observations of a SPC). Therefore, we suspect that the increasing relation between frequency of external mobilities and wage in Bowlus-Neumann[2004] is due to the composition effect.

2.3.2 Relation between wage earnings and frequency of promotions

Our empirical study then exceeds the analysis of Bowlus-Neumann[2004] by analyzing the impact of wage earnings on the probability of different types of job-to-job mobility. In the first instance, we consider the relation between frequency of promotion and wage earnings.

The figure 2.10 plots the frequency of promotions (internal and external) by wage decile for three SPC of interest. Because the SPC “managers” is already on the highest rung of the occupation hierarchy, the managers then will not be “promoted” by definition. This figure clearly indicates that, for all three SPC in consideration, the probability of promotion is always increasing with wage earnings. In other words, a higher-wage worker is more likely to be promoted than a lower-wage worker. Further, if we only consider the flows of promotion going with wage increase, the positively sloped profile of wage-promotion relation remains valid for the SPC “intermediate professions” and “skilled manual workers”.

By construction, the total flows of promotion consist of flows of internal promotion (intra-firm) and flows of external promotion (inter-firms). Indeed, because the flows of internal promotion accounts for 90% of total flows of
2.3. WAGE-TRANSITION RELATION

Figure 2.10: Frequency of promotions by wage decile

![Promotion Rate by Wage Decile](image)

- Total Promotions
- Promotions with wage increase

Figure 2.11: Frequency of job-to-job transitions by wage decile

![Transition Rate by Wage Decile](image)

- Total Transitions
- Transitions with wage increase

Manager
Intermediate Profession
Skilled Manual Worker
Unskilled Manual Worker
promotion, the observed relation between frequency of promotion and wage earnings is strongly dominated by the relation between frequency of internal promotion and wage earnings. On the other side, we also find that the frequency of external mobility is generally increasing with wage earnings, though this relation is not as strong as the relation between internal promotion and wage earnings.

2.3.3 Relation between wage earnings and frequency of total job-to-job transitions

As the total flows of job-to-job transitions consist of flows of promotion and flows of external mobility, we turn to consider, in this subsection, the impact of wage earnings on the frequency of job-to-job transitions.

Figure 2.11 plots the results for each one of four SPC in consideration. This figure gives several information of interest. First, the relation between the frequency of job-to-job transitions and wage earnings is rather ambiguous, typically first decreasing then increasing. However, if we only consider transitions that are associated with wage increases, the frequency of these mobilities tends to decrease with wage earnings. Further, the frequency of transitions associated with wage decreases, represented in the figure by the gap between the solid line and the dashed line, appears to be increasing with wage earnings.

Therefore, a joint examination of the figures 2.8, 2.10 and 2.11 indicates that the ambiguous relation between frequency of job-to-job transition and wage earnings refers to two particular types of job-to-job transition, they are, promotion and job-to-job transition associated with wage decrease.

The descriptive statistics presented in this section can already lead to three elementary conclusions:

- In the decade 1990-1999, around 40% to 55% of job-to-job transitions result in lower wages in new jobs. The literature is just beginning to develop models that have as a feature job changes with negative wage changes\(^7\). However, whether and to what extent these models allow to

\(^7\)Examples of models that allow to explain this particular type of mobility include
2.3. WAGE-TRANSITION RELATION

capture the actual relation between this particular type of mobility and the wage remains an open question.

• In this decade, around two thirds of job-to-job transitions are in nature promotion in sense of a raise of occupational category (SPC). Further, around 90% of promotions are made within firms, whereas only 10% are made via a change of firm. Therefore, internal promotions account for around 60% of total job-to-job transitions. Nevertheless, most equilibrium search models don’t allow to deal with this particular type of professional mobility.

• The frequency of external mobilities is unambiguously decreasing with wage earnings for any SPC in consideration. This empirical evidence confirms the prediction of equilibrium search theory that external mobility is a decreasing function of wage.

The frequency of promotions is always increasing with wage earnings. In other words, a higher-wage worker is more likely to be promoted than a lower-wage worker.

The relation between frequency of job-to-job transitions and wage earnings is \( U \)-shaped, typically first decreasing then increasing.

2.4 Empirical performance of the canonical equilibrium search model

In this section, we aim to compare the canonical equilibrium search model with the empirical data along two dimensions: wage dispersion and job-to-job transition. For this objective, we proceed to structurally estimate

the Burdett-Mortensen[1998] model with continuous productivity dispersion. This model is shown to allow for a close fit to the wage data (see Bontemps-Robin-Van den Berg[2000]).

As the flows of promotion are not considered in Burdett-Mortensen[1998], we will principally compare the model with the actual flows data about external mobilities that result in neither a raise of SPC nor a wage decline.

2.4.1 Relevant literature

Bowlus-Kiefer-Neuman[1995] provide the first empirical analysis of the Burdett-Mortensen[1998] model in which exogenous heterogeneity in employer productivity is allowed\(^8\). They assume only a finite number of employer types, each defined by a different value of productivity in the formal model. Because the model’s solution cannot be expressed in closed form in this case, their maximum likelihood estimation procedure becomes rapidly so computationally intensive that it is hardly possible to estimate a model with more than a few points of support for the distribution of productivity, while the fit to the wage data density of a model with a few points of support is not very satisfactory.

Bontemps-Robin-Van den Berg[2000] estimate the Burdett-Mortensen[1998] model while allowing for a continuous distribution of firm productivity types. To avoid problems of computational complexity encountered by Bowlus-Kiefer-Neuman[1995], the authors develop a structural nonparametric estimation method for the productivity distribution. Specially, Bontemps-Robin-Van den Berg[2000] use a kernel estimator and the data on employed worker earnings to fit the earnings distribution predicted by the model. Note, however, that their data about external mobilities includes some types of mobility that are not supported by the Burdett-Mortensen[1998] model, these including mobilities that result in a wage decline or a change of SPC.

The data sources used in Bontemps-Robin-Van den Berg[2000] are from

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2.4. STRUCTURAL ESTIMATION

French Labor Survey (FLS 1990-1993). After stratifying the data by industry, the authors find that the Burdett-Mortensen [1998] model with continuous productivity dispersion fits the wage data generally well, even though workers in each industry are assumed equally productive. To check the accuracy of the model in reproducing productivity data, Bontemps-Robin-Van den Berg [2000] compare the productivity distribution derived from the model with the actually observed productivity distribution issued from an independent data source, the French accounting firm data (BIC) \(^9\). The authors use the value added per worker as the measure of the productivity of the firm, then again the kernel estimator to generate a continuous distribution of productivity types for each industry considered. The authors find that, despite of some clear discrepancies between these two distributions, the predicted productivity distribution issued from the individual data (FLS) is reasonably close to that estimated from the firm data (BIC).

Bontemps-Robin-Van den Berg [2000] use a simpler, but faster and more powerful, nonparametric estimation method than the computationally intensive maximum likelihood estimation method as in Bowlus-Kiefer-Neuman [1995]. In what follows, we turn to use a simulated method of moments to estimate the Burdett-Mortensen [1998] model with continuous distribution of firm productivity types. Our work distinguishes from the above authors mainly in its objective: we seek for test the capacity of this model in reproducing not only actual wage or productivity distribution, but also flows data about job-to-job transitions of individuals. The latter is a dimension until now largely neglected in the literature.

2.4.2 Estimation method

As well known, the maximum likelihood (ML) estimator is fully efficient among consistent and asymptotically normally distributed estimators, in the context of the specified parametric model. The possible shortcoming in this result is that to attain that efficiency, it is necessary to make possibly strong, restrictive assumptions about the functional form of the distribution. The generalized method of moments (GMM) estimators move away from paramet-

\(^9\)FLS databases contain extensive individual information, but no firm information.
ric assumptions, toward estimators which are robust to some variations in the underlying data generating process.

With regard to the GMM estimator, the estimation of unknown parameters involves optimizing a criterion function based on a set of moment restrictions. Unfortunately, for many econometric models the relevant moment restrictions do not have a tractable analytical form in terms of the unknown parameters rendering the estimation by the generalized method of moments infeasible. In contrast, the simulated method of moments (SMM) estimator modifies the traditional GMM estimator by using moments computed from simulated data of the model rather than the analytical moments. Like the GMM estimator, the SMM estimator is consistent and asymptotically normal when the number of observations tends to infinity, and is asymptotically equivalent to GMM if the number of simulations approaches infinity. In what follows, the SMM estimator is implemented to estimate the Burdett-Mortensen[1998] model with continuous productivity distribution. See equally appendix C of chapter 4 for detailed presentation of the SMM estimator.

We consider the case where the productivity of the firm follows a Pareto distribution $\Gamma(p) = 1 - (p_{\text{min}}/p)\beta$, $p_{\text{min}}$ is the lower bound of the support$^{11}$. The following vector $(\text{dim}(\Phi) = 7)$ synthesizes all the parameters of the model:

$$
\Phi = \{r, b, \delta, \lambda_0, \lambda_1, \beta, p_{\text{min}}\}
$$

Market equilibrium in Burdett-Mortensen[1998] can be summarized by a distribution of wage earnings $G(w)$, a distribution of wage offers $F(w)$ and a

---


$^{11}$As pointed out by Bontemps-Robin-Van den Berg[2000], the estimated productivity densities derived from the non-parametric estimation procedure resemble Pareto densities for a wide range of values and for most industries. Our estimation is less ambitious than theirs. A less exigent accuracy in conformity with productivity data is admissible.
wage function conditional on productivity of the firm $w(p)$. We restate here the equilibrium conditions:

\[
\begin{align*}
G'(w(p)) &= \frac{f(w(p))(1 + k_1)}{[1 + k_1(1 - F(w(p)))]^2} \\
F'(w(p)) &= \frac{1 + k_1(1 - \Gamma(p))}{2k_1[p - w(p)]} \\
w'(p) &= \Gamma'(p) \frac{2k_1[p - w(p)]}{1 + k_1(1 - \Gamma(p))}
\end{align*}
\]

where $k_1 = \lambda_1/\delta$. The probability of job-to-job transition is then defined by $m(w(p)) = \lambda_1[1 - \Gamma(p)]$.

Note that these three functions are fully determined by the productivity distribution $\Gamma(p)$, and by two transition parameters $\delta$ and $\lambda_1$. To simplify the estimation procedure, we assume that wage offers are bounded by a mandatory minimum wage which satisfies $w_{\text{min}} \geq R$ where $R$ is the reservation wage of the unemployed. Therefore three parameters, $\{r, b, \lambda_0\}$, will have no incidence to the market equilibrium\(^\text{12}\). By this way, we restrict the parameters to be estimated to

\[
\theta = \{\delta, \lambda_1, \beta, p_{\text{min}}\}
\]

where $\dim(\theta) = 4$. $\theta$ denotes the vector of unknown structural parameter.

We use data issued from French labor survey (FLS 1990-1999), see section 2.2 for presentation of data. We run estimation separatively for each of four occupational categories (SPC) retained in the sample. Within each SPC, the lowest wage observed is normalized to be unity\(^\text{13}\). The moments underlying the estimation are based on the wage distribution. We calculate the mean wage within each wage decile, denoted by $w(D_i)$ where $i = 1 \ldots 10$, and the mean wage within each SPC, denoted by $\bar{w}$. The subsequent estimations are performed basing on these eleven moments. Formally, the SMM estimator is implemented as follows:

\(^{12}\)After estimation, we have verified that there existed a vector $\{r, b, \lambda_0\}$ that guaranteed $R \leq w_{\text{min}}$ given the estimates of other parameters.

\(^{13}\)The lowest wage observed within each SPC is equal or greater than the mandatory minimum wage rate of every year over this decade.
Step 1 Calculate a 11-dimensional vector of moment, $M_{data}$, from the data.

$$
\mathcal{M} = [w(D1), w(D2), \ldots, w(D10), \tilde{w}]
$$

where $D1, \ldots, D10$ denote the wage deciles. This set of moments aims to test the capacity of the model in precisely capturing the shape of actual wage distribution.

Step 2 Given the vector of unknown structural parameter, $\theta$, a corresponding set of simulated moments, $M_{model}$, is calculated from the structural model.

Step 3 A SMM estimate $\hat{\theta}_{SMM}$ for $\theta$ minimizes an objective function $Q$ of quadratic form:

$$
Q = \min_\theta f(\theta) = \min_\theta D'WD
$$

where $D = (M_{model} - M_{data})$, $W$ is a symmetric non-negative definite weighting matrix defining the metric\textsuperscript{14}. Steps 2 and 3 are conducted until convergence, i.e. until an estimate $\hat{\theta}_{SMM}$ that minimizes the objective function is obtained\textsuperscript{15}.

To examine the empirical performance of the model, we first perform the usual t-tests based on confidence intervals. This sort of hypothesis tests permits to detect whether the vector of unknown parameter, the actual and simulated moments are all precisely evaluated. Next, looking through the moments one by one, we seek to detect how much the moments generated by the model coincide with those observed in the data. For any given moment in the vector $D = (M_{model} - M_{data})$, a smaller value indicates that the structural model is able to account for this specific feature of the data, while a larger

\textsuperscript{14}This matrix is given by the inverse of the variance-covariance matrix of the moments obtained from actual data, $W = \left\{ \text{Asy.Cov}(\sqrt{N}M_{data}) \right\}^{-1}$ where $N$ is the sample size.

\textsuperscript{15}The minimization of the objective function is performed by way of utilities provided by MATLAB Optimization Toolbox. MATLAB \textit{fminsearch} finds a minimum of a scalar function of several variables, starting at an initial estimate. This is generally referred to as unconstrained nonlinear optimization.
value may reveal some failures. This leads us to perform a moment-specific diagnostic test. The first order condition associated to the minimization of the objective function $Q$ requires:

$$\left( \frac{\partial D}{\partial \theta} \right)_{\theta=\hat{\theta}_{SMM}} \bigg| WD = [0]$$

Let $G$ denote the gradient matrix $G = \frac{\partial D}{\partial \theta}$. Using the mean value approximation of $G$, one constructs the statistic

$$T = \left\{ \text{diag} \left[ W^{-1} - G (G'WG)^{-1} G' \right] \right\}^{-1/2} \sqrt{ND}$$

where $N$ is the sample size. Each element of the vector $T$ follows asymptotically a standard normal distribution $N(0, 1)$\(^{16}\).

Following Hansen\cite{Hansen82}, we perform finally a global specification test to see whether the selected set of moments is generally accepted by the data. The related statistic, denoted by $J = ND WD$, is asymptotically distributed as a chi-square, with a degree of freedom equal to the number of overidentifying restrictions.

### 2.4.3 Estimation results

The estimation results are reported in detail in the tables 2.4-2.7 in appendix A. Giving these tables a glance before further discussion, we find that all the unknown structural parameters are precisely estimated. Moreover, that both actual and simulated moments are significantly different from zero makes the set of moments an exigent criterion to test the model’s capacity in reproducing actual wage distribution. Looking through the moments one by one, the simulated moments well match their empirical counterpart for every occupational category (SPC). The sole exception is the mean wage of the last decile, $w(D10)$, that the model generally fails to replicate. Therefore, our estimation confirms the conclusion of Bontemps-Robin-Van den Berg\cite{Bontemps00} that the Burdett-Mortensen\cite{Burdett98} model with continuous productivity distribution provides, in general, a reasonably close fit to the actual wage data.

The estimates of two transition parameters, $\delta$ and $\lambda_1$, are comparable to those of Bontemps-Robin-Van den Berg\cite{Bontemps00} and Postel-Vinay and

\(^{16}\)See Collard et al\cite{Collard02} for more details.
Table 2.3: Estimates of transition parameters (annual values)

<table>
<thead>
<tr>
<th>Authors</th>
<th>$\delta$</th>
<th>$\lambda_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bontemps et al.</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All sample</td>
<td>0.0737</td>
<td>0.0953</td>
</tr>
<tr>
<td>Food</td>
<td>0.0763</td>
<td>0.0736</td>
</tr>
<tr>
<td>Intermediary goods</td>
<td>0.0597</td>
<td>0.0738</td>
</tr>
<tr>
<td>Equipment</td>
<td>0.0580</td>
<td>0.0696</td>
</tr>
<tr>
<td>Current consumption</td>
<td>0.0783</td>
<td>0.0660</td>
</tr>
<tr>
<td>Construction</td>
<td>0.0905</td>
<td>0.0871</td>
</tr>
<tr>
<td>Trade</td>
<td>0.0950</td>
<td>0.0950</td>
</tr>
<tr>
<td>Transport</td>
<td>0.0557</td>
<td>0.1387</td>
</tr>
<tr>
<td>Service</td>
<td>0.1037</td>
<td>0.1456</td>
</tr>
<tr>
<td><strong>Postel-Vinay et al.</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Executives, managers and engineers</td>
<td>0.0776</td>
<td>0.643</td>
</tr>
<tr>
<td>Supervisors, administrative and sales</td>
<td>0.0859</td>
<td>0.666</td>
</tr>
<tr>
<td>Technical supervisors and technicians</td>
<td>0.0686</td>
<td>0.646</td>
</tr>
<tr>
<td>Administrative support</td>
<td>0.0932</td>
<td>0.737</td>
</tr>
<tr>
<td>Skilled manual workers</td>
<td>0.0886</td>
<td>0.685</td>
</tr>
<tr>
<td>Sales and service workers</td>
<td>0.1016</td>
<td>0.716</td>
</tr>
<tr>
<td>Unskilled manual workers</td>
<td>0.0989</td>
<td>0.666</td>
</tr>
<tr>
<td><strong>This Study</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Managers</td>
<td>0.0109</td>
<td>0.1262</td>
</tr>
<tr>
<td>Intermediate Professions</td>
<td>0.0190</td>
<td>0.2304</td>
</tr>
<tr>
<td>Skilled manual workers</td>
<td>0.0665</td>
<td>0.2645</td>
</tr>
<tr>
<td>Unskilled manual workers</td>
<td>0.0761</td>
<td>0.1966</td>
</tr>
</tbody>
</table>

Source: Bontemps-Robin-Van den Berg[2000], table 2; Postel-Vinay and Robin[2002], table IV.
Robin[2002], these authors use also French panel data to effectuate their estimation\textsuperscript{17}. Note that Bontemps-Robin-Van den Berg[2000] estimate the same Burdett-Mortensen[1998] model as us but stratify the data by industry; in contrast, Postel-Vinay and Robin[2002] estimate their own equilibrium search model but stratify the data by SPC (See subsection 1.6.1 in chapter 1 for presentation of the model of Postel-Vinay and Robin[2002]).

The table 2.3 gives a comparison between their estimates of these two transition parameters and the ours. With regard to the job contact rate, our estimates of $\lambda_1$ is generally above 0.2 for SPC “intermediate professions”, “skilled workers” and “unskilled workers”, which constitute some intermediate values between estimates of Bontemps-Robin-Van den Berg[2000] ($\lambda_1$ around 0.07-0.14) and those of Postel-Vinay and Robin[2002] ($\lambda_1$ larger than 0.6). If we turn to examine the job destruction rate, we find that our estimates of $\delta$ for SPC “skilled workers” and “unskilled workers” are generally the same as those in Postel-Vinay and Robin[2002] and Bontemps-Robin-Van den Berg[2000]: all around 0.07. Interesting, the estimates of $\delta$ for another two SPC “managers” and “intermediate professions”, on the order of 0.01-0.02, are rather small. This result implies that the flows of job destructions are markedly lower among the managers, especially comparing to the flows of job destruction in the SPC “workers”.

The figure 2.12 compares, for each of four SPC, the Burdett-Mortensen[1998] model with the empirical data along two dimensions: wage dispersion and job-to-job transition. From this figure, the observed bell shape of wage data density is particularly well replicated by the model, but not the flat right tail. This point is particularly emphasized by Bontemps-Robin-Van den Berg[2000]. It is this insufficiency that explicates the relatively higher value of the $J$ statistic leading to globally reject the model at the 10\% significance level\textsuperscript{18}. However moment by moment, the model is generally accepted.

This figure gives also an intuitive comparison between the wage-mobility


\textsuperscript{18}Our main objective of this study is not to validate or de-validate the model. Bontemps-Robin-Van den Berg[2000] and us all find that, generally speaking, the Burdett-Mortensen[1998] model allows for a reasonably close fit to the wage data, but the coincidence is not perfect.
CHAPTER 2. EMPIRICAL EVIDENCES

Figure 2.12: $g(w)$ and $m(w)$ by SPC

“Manager”

“Intermediate profession”

“Skilled manual worker”

“Unskilled manual worker”
relation predicted by the model and that observed in the actual data. As the flows of promotion are not considered in Burdett-Mortensen[1998], we compare the model with the actual flows data about external mobilities that don’t result in a raise of SPC. It’s interesting to remark that the Burdett-Mortensen[1998] model permits to account for the observed negative relation between wage earnings and external mobility. This result becomes more evident if we only consider the flows of external mobility that result in wage increases. When the flows of external mobility that go with wage decreases are equally integrated, the observed wage-transition profile is much smoother than the predicted one.

2.5 Conclusion

Most equilibrium search models predict a negative association between wage and probability of external mobility. Using U.S. data, Bowlus-Neumann[2004] find that the frequency of external mobilities is generally increasing with wages decile. Using French data, we classify the individuals by their occupational category (Socio-Professional Category, or SPC). We show that the frequency of external mobilities is unambiguously decreasing with wage decile for any SPC in consideration. However at the aggregate level, i.e., if we don’t classify the individuals by SPC, the relation between frequency of external mobilities and wage is rather ambiguous. We then suspect that the increasing relation between frequency of external mobilities and wage decile in Bowlus-Neumann[2004] is due to a composition effect. Our empirical investigation then confirms the validity of equilibrium search theory in this dimension while providing additional insights into the actual relation between wage earnings and job-to-job transitions.

We decompose the total flows of job-to-job transition into flows of promotion and flows of external mobility. We also distinguish, among the flows of job-to-job transition, the job changes associated with wage increase from those with wage cut. We show that most of equilibrium search models fail to capture a substantial part of total job-to-job transitions: this notably includes mobilities associated with wage decreases (around 40-55%). Furthermore, the question of promotions is not directly discussed by these models,
whereas promotions account for around two thirds in the total job-to-job transitions.

We find that the frequency of promotions is always increasing with wage earnings. In other words, a higher-wage worker is more likely to be promoted than a lower-wage worker. The relation between frequency of job-to-job transitions and wage earnings is \( U \)-shaped, typically first decreasing then increasing. This \( U \)-shaped relation refers to two particular types of job-to-job transition, they are, promotion and job-to-job transition associated with wage decrease. Therefore, our empirical findings don’t oppugn the validity of the equilibrium search theory, as these two types of job-to-job transition are not explicitly considered by the theory. An extension of the equilibrium search model is necessary and permits to predict more realistic wage and job trajectories of individuals in the labor market.

Appendix A: Tables of estimation results
Table 2.4: Estimation Results for SPC “Unskilled Manual Worker”

<table>
<thead>
<tr>
<th>Over-identification test (J-test)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.0279</td>
<td>2.1670</td>
<td>0.1176</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Estimates of parameters</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>( \hat{\theta}_{SMM} )</td>
<td>s.d.</td>
<td>( t - \text{Stat.}^{\dagger} )</td>
<td>Prob.</td>
</tr>
<tr>
<td>( \beta )</td>
<td>4.4310***</td>
<td>0.2216</td>
<td>19.9995</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \rho_{\text{min}} )</td>
<td>1.1971***</td>
<td>0.0219</td>
<td>54.7162</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \lambda_1 )</td>
<td>0.1966***</td>
<td>0.0092</td>
<td>21.3317</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.0761***</td>
<td>0.0184</td>
<td>4.1277</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Estimates of moments</th>
<th>( M_{\text{model}} )</th>
<th>( M_{\text{data}} )</th>
<th>( t - \text{Stat.}^{\dagger} )</th>
<th>Accept.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w(D1) )</td>
<td>1.0464*** (0.0306)</td>
<td>1.0627*** (0.0269)</td>
<td>0.5341</td>
<td>1</td>
</tr>
<tr>
<td>( w(D2) )</td>
<td>1.1357*** (0.0209)</td>
<td>1.1427*** (0.0204)</td>
<td>0.3370</td>
<td>1</td>
</tr>
<tr>
<td>( w(D3) )</td>
<td>1.2065*** (0.0189)</td>
<td>1.2102*** (0.0185)</td>
<td>0.1957</td>
<td>1</td>
</tr>
<tr>
<td>( w(D4) )</td>
<td>1.2710*** (0.0173)</td>
<td>1.2708*** (0.0189)</td>
<td>0.0107</td>
<td>1</td>
</tr>
<tr>
<td>( w(D5) )</td>
<td>1.3371*** (0.0199)</td>
<td>1.3357*** (0.0187)</td>
<td>0.0671</td>
<td>1</td>
</tr>
<tr>
<td>( w(D6) )</td>
<td>1.4097*** (0.0214)</td>
<td>1.4001*** (0.0210)</td>
<td>0.4564</td>
<td>1</td>
</tr>
<tr>
<td>( w(D7) )</td>
<td>1.4851*** (0.0217)</td>
<td>1.4779*** (0.0239)</td>
<td>0.3320</td>
<td>1</td>
</tr>
<tr>
<td>( w(D8) )</td>
<td>1.5718*** (0.0279)</td>
<td>1.5785*** (0.0339)</td>
<td>0.2422</td>
<td>1</td>
</tr>
<tr>
<td>( w(D9) )</td>
<td>1.6796*** (0.0339)</td>
<td>1.7259*** (0.0535)</td>
<td>1.3654</td>
<td>1</td>
</tr>
<tr>
<td>( w(D10) )</td>
<td>1.9042*** (0.0691)</td>
<td>2.1020*** (0.2387)</td>
<td>2.8645</td>
<td>0</td>
</tr>
<tr>
<td>( \bar{w} )</td>
<td>1.6690*** (0.2842)</td>
<td>1.4306*** (0.3045)</td>
<td>0.8391</td>
<td>1</td>
</tr>
</tbody>
</table>

Legend: * Significant at 10% \((t - \text{test})\),
** Significant at 5%, *** Significant at 1%.

\( H_0: \hat{\theta}_{SMM} = 0 \) or \( M_{\text{model}|\text{data}} = 0 \),
\( \dagger H_0: M_{\text{model}} = M_{\text{data}} \).

Accept. = 1 if \( t - \text{Stat.}^{\dagger} < 1.96 \), Accept. = 0 otherwise.
### Table 2.5: Estimation Results for SPC “Skilled Manual Worker”

**Over-identification test (J-test)**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$J$</td>
<td>2.9554</td>
<td>2.1670</td>
</tr>
</tbody>
</table>

**Estimates of parameters**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>4.4113***</td>
<td>0.5216</td>
<td>8.4570</td>
<td>0.0000</td>
</tr>
<tr>
<td>$p_{\min}$</td>
<td>1.2888***</td>
<td>0.0287</td>
<td>44.8773</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.2645***</td>
<td>0.0130</td>
<td>20.3420</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.0665***</td>
<td>0.0073</td>
<td>9.1454</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

**Estimates of moments**

<table>
<thead>
<tr>
<th>Moment</th>
<th>$M_{\text{model}}$</th>
<th>$M_{\text{data}}$</th>
<th>t Stat.‡</th>
<th>Accept.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w(D1)$</td>
<td>1.0978*** (0.0588)</td>
<td>1.1322*** (0.0550)</td>
<td>0.5854</td>
<td>1</td>
</tr>
<tr>
<td>$w(D2)$</td>
<td>1.2547*** (0.0350)</td>
<td>1.2686*** (0.0300)</td>
<td>0.3952</td>
<td>1</td>
</tr>
<tr>
<td>$w(D3)$</td>
<td>1.3554*** (0.0221)</td>
<td>1.3606*** (0.0250)</td>
<td>0.2357</td>
<td>1</td>
</tr>
<tr>
<td>$w(D4)$</td>
<td>1.4446*** (0.0271)</td>
<td>1.4459*** (0.0245)</td>
<td>0.0462</td>
<td>1</td>
</tr>
<tr>
<td>$w(D5)$</td>
<td>1.5360*** (0.0243)</td>
<td>1.5296*** (0.0244)</td>
<td>0.2645</td>
<td>1</td>
</tr>
<tr>
<td>$w(D6)$</td>
<td>1.6275*** (0.0272)</td>
<td>1.6165*** (0.0262)</td>
<td>0.4054</td>
<td>1</td>
</tr>
<tr>
<td>$w(D7)$</td>
<td>1.7255*** (0.0284)</td>
<td>1.7167*** (0.0310)</td>
<td>0.3111</td>
<td>1</td>
</tr>
<tr>
<td>$w(D8)$</td>
<td>1.8367*** (0.0350)</td>
<td>1.8439*** (0.0412)</td>
<td>0.2060</td>
<td>1</td>
</tr>
<tr>
<td>$w(D9)$</td>
<td>1.9749*** (0.0440)</td>
<td>2.0290*** (0.0691)</td>
<td>1.2321</td>
<td>1</td>
</tr>
<tr>
<td>$w(D10)$</td>
<td>2.2665*** (0.0874)</td>
<td>2.4864*** (0.2821)</td>
<td>2.5142</td>
<td>0</td>
</tr>
<tr>
<td>$\tilde{w}$</td>
<td>1.9769*** (0.3646)</td>
<td>1.6425*** (0.3960)</td>
<td>0.9174</td>
<td>1</td>
</tr>
</tbody>
</table>

**Legend:** * Significative at 10% ($t - \text{test}$), ** Significative at 5%, *** Significative at 1%.

‡ $H_0$: $\theta_{\text{SMM}} = 0$ or $M_{\text{model}}|_{\text{data}} = 0$, † $H_0$: $M_{\text{model}} = M_{\text{data}}$. Accept. = 1 if $t - \text{Stat.} \leq 1.96$, Accept. = 0 otherwise.
Table 2.6: Estimation Results for SPC “Intermediate Profession”

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Over-identification test (J-test)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J$ Stat.</td>
<td>3.0858</td>
<td>2.1670</td>
<td>0.1230</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Estimates of parameters</th>
<th>$\theta$</th>
<th>$\hat{\theta}_{SMM}$</th>
<th>s.d.</th>
<th>$t$ – Stat.†</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>4.0812***</td>
<td>0.1079</td>
<td></td>
<td>37.8290</td>
<td>0.0000</td>
</tr>
<tr>
<td>$p_{min}$</td>
<td>1.3842***</td>
<td>0.0222</td>
<td></td>
<td>62.3062</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.2304***</td>
<td>0.0111</td>
<td></td>
<td>20.6963</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.0190***</td>
<td>0.0008</td>
<td></td>
<td>23.7479</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Estimates of moments</th>
<th>$M_{model}$</th>
<th>$M_{data}$</th>
<th>$t$ – Stat.‡</th>
<th>Accept.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w(D1)$</td>
<td>1.3070***</td>
<td>1.3726***</td>
<td>0.4837</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(0.1358)</td>
<td>(0.1276)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w(D2)$</td>
<td>1.6110***</td>
<td>1.6467***</td>
<td>0.5506</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(0.0647)</td>
<td>(0.0565)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w(D3)$</td>
<td>1.8046***</td>
<td>1.8176***</td>
<td>0.2708</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(0.0478)</td>
<td>(0.0428)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w(D4)$</td>
<td>1.9660***</td>
<td>1.9609***</td>
<td>0.1144</td>
<td>1</td>
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<tr>
<td></td>
<td>(0.0440)</td>
<td>(0.0419)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w(D5)$</td>
<td>2.1149***</td>
<td>2.0991***</td>
<td>0.3888</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(0.0405)</td>
<td>(0.0409)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w(D6)$</td>
<td>2.2643***</td>
<td>2.2489***</td>
<td>0.3485</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(0.0444)</td>
<td>(0.0446)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w(D7)$</td>
<td>2.4330***</td>
<td>2.4195***</td>
<td>0.2600</td>
<td>1</td>
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<tr>
<td></td>
<td>(0.0520)</td>
<td>(0.0518)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w(D8)$</td>
<td>2.6260***</td>
<td>2.6327***</td>
<td>0.1148</td>
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<tr>
<td></td>
<td>(0.0587)</td>
<td>(0.0688)</td>
<td></td>
<td></td>
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<tr>
<td>$w(D9)$</td>
<td>2.8492***</td>
<td>2.9465***</td>
<td>1.4032</td>
<td>1</td>
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<tr>
<td></td>
<td>(0.0693)</td>
<td>(0.1154)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w(D10)$</td>
<td>3.3376***</td>
<td>3.7493***</td>
<td>2.5902</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(0.1590)</td>
<td>(0.5132)</td>
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<td></td>
</tr>
<tr>
<td>$\tilde{w}$</td>
<td>2.7974***</td>
<td>2.2888***</td>
<td>0.7991</td>
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</tr>
<tr>
<td></td>
<td>(0.6364)</td>
<td>(0.7040)</td>
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<td></td>
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</tbody>
</table>

Legend: * Significative at 10% ($t$ – test†), ** Significative at 5%, *** Significative at 1%.
† $H_0$: $\hat{\theta}_{SMM} = 0$ or $M_{model}|data = 0$, ‡ $H_0$: $M_{model} = M_{data}$.
Accept. = 1 if $t$ – Stat.‡ < 1.96, Accept. = 0 otherwise.
Table 2.7: Estimation Results for SPC “Manager”

<table>
<thead>
<tr>
<th>Over-identification test (J-test)</th>
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<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>J - Stat</td>
<td>Crit. Stat</td>
<td>Prob</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.9975</td>
<td>2.1670</td>
<td>0.3397</td>
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</table>

<table>
<thead>
<tr>
<th>Estimates of parameters</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>( \hat{\theta}_{SMM} )</td>
<td>s.d.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta )</td>
<td>3.2325***</td>
<td>0.2092</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p_{min} )</td>
<td>2.0528***</td>
<td>0.1411</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \lambda_1 )</td>
<td>0.1262***</td>
<td>0.0125</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.0109***</td>
<td>0.0006</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Estimates of moments</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment</td>
<td>( M_{model} )</td>
<td>( M_{data} )</td>
<td>t - Stat</td>
<td>Accept.</td>
</tr>
<tr>
<td>( w(D1) )</td>
<td>1.7797***</td>
<td>2.0050***</td>
<td>0.7227</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(0.3117)</td>
<td>(0.2793)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w(D2) )</td>
<td>2.4261***</td>
<td>2.5191***</td>
<td>0.6959</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(0.1337)</td>
<td>(0.1003)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w(D3) )</td>
<td>2.8185***</td>
<td>2.8430***</td>
<td>0.2516</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(0.0973)</td>
<td>(0.0812)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w(D4) )</td>
<td>3.1416***</td>
<td>3.1197***</td>
<td>0.2485</td>
<td>1</td>
</tr>
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<td></td>
<td>(0.0880)</td>
<td>(0.0836)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w(D5) )</td>
<td>3.4596***</td>
<td>3.4143***</td>
<td>0.4831</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(0.0938)</td>
<td>(0.0860)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w(D6) )</td>
<td>3.7912***</td>
<td>3.7341***</td>
<td>0.5928</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(0.0963)</td>
<td>(0.1026)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w(D7) )</td>
<td>4.1591***</td>
<td>4.1162***</td>
<td>0.3732</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(0.1150)</td>
<td>(0.1264)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w(D8) )</td>
<td>4.5674***</td>
<td>4.6242***</td>
<td>0.4732</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(0.1201)</td>
<td>(0.1737)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w(D9) )</td>
<td>5.0757***</td>
<td>5.3894***</td>
<td>1.8271</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(0.1717)</td>
<td>(0.2913)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w(D10) )</td>
<td>6.1733***</td>
<td>7.2319***</td>
<td>2.7683</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(0.3824)</td>
<td>(1.1479)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \bar{w} )</td>
<td>4.9111***</td>
<td>3.8979***</td>
<td>0.7382</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(1.3725)</td>
<td>(1.5965)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Legend: * Significative at 10\% (t - test),
** Significative at 5\%, *** Significative at 1\%.
\( \hat{\theta}_{SMM} = 0 \) or \( M_{model} \mid data = 0 \), \( H_0: M_{model} = M_{data} \).
Accept. = 1 if \( t - Stat \)\( \dagger \) < 1.96, Accept. = 0 otherwise.
Chapter 3

An equilibrium search model with two types of jobs: the role of endogenous search effort reinspected

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CHAPTER 3. ENDOGENOUS SEARCH EFFORT REINSPECTED
3.1 Introduction

The work of Burdett-Mortensen[1998] pioneers modern research of the labor market using equilibrium search framework. Burdett-Mortensen[1998] indeed show that if the employees are also allowed to search on-the-job for better outside alternative, a non-degenerate wage distribution is a robust equilibrium outcome even though all firms and workers are respectively identical ex ante.

Nevertheless, the theory of pure wage dispersion of Burdett-Mortensen[1998] typically predicts that the equilibrium wage density is monotonically increasing in wages in apparent contradiction with actual shape of wage distribution. So in order to match the observed wage dispersion for identical workers, usually we need some form of heterogeneity as in Bontemps-Robin-Van den Berg[2000], Mortensen[1998] or Postel-Vinay and Robin[2002](see chapter 1 for a survey of these work). However, we have not yet a satisfactory explanation of wage dispersion in a pure environment where all workers and firms are respectively identical.

Further, the assumption of endogenous search effort in the basic Burdett-Mortensen[1998] framework aggravates the difficulty of the model in matching the observed wage dispersion(see Mortensen[2002b]). Let \( s(w) \) denote the search effort of the worker conditional on the wage rate. Because the likelihood of finding a better job declines with the wage earned, search intensity declines with an employed worker’s current wage, i.e., \( s'(w) < 0 \) associated with the boundary condition \( s(\bar{w}) = 0 \). This equilibrium property even induces a stronger decrease of the probability of job-to-job transition when the wage increases, and gives firms additional incentive to post high wages. Therefore as in the basic Burdett-Mortensen[1998] model, the equilibrium wage density is again monotonically increasing in wages.

We should point out that the flow of job-to-job transition from one occupational category (SPC) to another is a dimension up to now largely neglected. Importantly, we argue that extending the equilibrium search model to account for this dimension is a fruitful way to reinspect the shape of endogenous search effort \( s(w) \), hence that of equilibrium wage distribution.
We thus construct a job search model with two types of jobs. Homogenous workers start working in jobs with low productivity. They face an exogenous risk of unemployment, and search on-the-job not only for better opportunities in low productivity jobs, but also for acceding to high productivity jobs. We also assume that employees in high-productivity jobs segment could only be recruited from low-productivity jobs segment, they stop searching on-the-job and face no more firing risk. In this circumstance, a transition from a low-productivity job to a high-productivity job refers to a promotion, whereas a transition from a low-productivity job to another low-productivity job refers to an external mobility (without promotion). Briefly speaking, the model describes a two-tier structure of labor market where employees in the upper segment are stable, but mobile in the lower segment.

In low-productivity jobs, the wage is posted by the firm. Once a worker accedes to a high productivity job, it is assumed that the wage is negotiated according to a sharing rule. This view of dual wage determination rules is supported by several empirical studies using French data. For example, Cahuc, Postel-Vinay and Robin[2006] emphasize that low-skilled workers are mainly related to on-the-job search and wage posting, whereas the impact of bargaining power is much more important for high-skilled workers (see table 3.1).

### Table 3.1: Estimates of bargaining power by occupational category (SPC)

<table>
<thead>
<tr>
<th>Industry</th>
<th>Bargaining power</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MAN</td>
<td>INT</td>
<td>SW</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.35</td>
<td>0.13</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Construction</td>
<td>0.98</td>
<td>0.26</td>
<td>0.15</td>
<td>0.17</td>
</tr>
<tr>
<td>Trade</td>
<td>0.38</td>
<td>0.33</td>
<td>0.14</td>
<td>0.00</td>
</tr>
<tr>
<td>Services</td>
<td>0.16</td>
<td>0.00</td>
<td>0.08</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Source: Cahuc, Postel-Vinay and Robin[2006], table 4.

The key characteristic of the model is that wage distribution within either jobs segment is bell-shaped. Note, particularly, that we obtain this result in a pure environment where all firms and workers are respectively identical. Let’s first consider the shape of wage dispersion in low-productivity jobs segment. In most equilibrium search models (including this one), a higher wage
3.1. INTRODUCTION

will reduce the probability that an employee receives wage increases in jobs segment with low productivity. This induces a positive relation between the duration of employment and the wage. However in view of the opportunity of job-to-job transition from a low-productivity job to a high-productivity job, the search effort of the employee may increase with the wage if the preferences between consumption and leisure are non-separable and if consumption and leisure are substitutable goods. Therefore, the effect of promotion induces a negative relation between the duration of employment and the wage, so represents an opposite force to the positive one. The optimal wage policy of the firm trades off these two competing effects. As we shall see, it leads to a market equilibrium where a bell-shaped wage density will be a typical equilibrium outcome.

Next, we consider the shape of wage density in high-productivity jobs segment. As it is assumed that the wage is negotiated between employee and employer in high-productivity jobs, the Nash solution to the bargaining game implies that the negotiated wage is indeed a weighted average of the productivity in a high productivity job and the reservation wage of the employee. Note that employees in high-productivity jobs segment could only be recruited from low-productivity jobs segment whereas they retire from the market at an exogenous rate $\mu$. Therefore, the wage distribution in high-productivity jobs segment is a transformation of wage distribution in low-productivity jobs segment. In other words, a bell-shaped wage density in low-productivity jobs segment will typically induce a bell-shaped wage density in high-productivity jobs segment. By this way, the model allows that wage distribution within either jobs segment is bell-shaped.

This chapter is organized as follows. Section 2 describes the model. Section 3 shows the simulation results. The last section concludes.

3.2 Model

We distinguish two jobs segments: low-productivity jobs segment and high-productivity jobs segment. This distinction refers to differences in occupational categories. Let $p_i$ be marginal productivity of efficient labor in occupational category $i$ where $i = 1, 2$, we assume $p_1 < p_2$. 
Homogenous workers start working in jobs with low productivity. They face an exogenous risk of unemployment denoted as $\delta$, and search on-the-job not only for better opportunities in low productivity jobs, but also for acceding to high productivity jobs. Search is sequential and non-directed. Assume that offers arrive at rate $\lambda_0$ when unemployed. Let $s(w)$ denote the search intensity which is a function of the wage. Let also $\lambda_1(s(w))$ and $\lambda_2(s(w))$ represent the arrival rates of offers from low-productivity jobs segment and high-productivity jobs segment, respectively. We assume that $\lambda_1(s(w))$ and $\lambda_2(s(w))$ are both increasing functions of search effort.

In low-productivity jobs, the wage is posted by the firm. Once a worker accedes to a high productivity job, it is assumed that the wage is negotiated according to a sharing rule. We also assume that employees in high-productivity jobs segment could only be recruited from low-productivity jobs segment. They stop searching on-the-job, face no more firing risk and retire from the market at rate $\mu$. In this circumstance, a transition from a low-productivity job to a high-productivity job refers to a promotion, whereas a transition from a low-productivity job to another low-productivity job refers to an external mobility (without promotion). Therefore, the model describes a two-tier structure of labor market where employees in the upper segment are stable, but mobile in the lower segment.

### 3.2.1 Labor market flows

Suppose a labor market in steady state where all workers are identical. Because all equilibrium wage offers are acceptable, unemployed workers find employment at rate $\lambda_0$. Let normalize to one the sum of unemployed workers and employees in low productivity jobs. In the steady state, the balance between flows into and out of unemployment assuming a constant unemployment rate is written as:

$$(1 - u)\delta = \lambda_0 u$$

where $\delta$ represents the job destruction rate.

Let $G_1$ and $F_1$ denote the cumulative distribution function of earnings and wage offers in low-productivity jobs segment, the stock of employees earning $w$ or less in low productivity jobs is $(1 - u)G_1(w)$. 
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Workers leave this stock because they are laid off (which happens at rate \( \delta \)), because they receive an outside offer of a low productivity job with associated wage greater than \( w \) (at rate \( (1 - F(w)) \int_w^w \lambda_1(s(y)) dG_1(y) \)), because they receive an offer of a high productivity job (at rate \( \int_w^w \lambda_2(s(y)) dG_1(y) \)), or finally because they are hit by a reallocation shock but lucky enough to draw a new wage greater than \( w \) (at rate \( \tilde{\lambda}_1(1 - F(w)) \)). The reallocation shock, which happened at rate \( \tilde{\lambda}_1 \), is equivalent to a layoff immediately followed by a job offer.

On the other side, the inflows into the stock \((1 - u)G_1(w)\) consist of unemployed workers who draw a wage offer below \( w \) (the measure of such entrants is \( u\lambda_0 F_1(w) \)), and employees who are previously employed at wage above \( w \) but hit by a reallocation shock and draw a new wage below \( w \) (the measure of such entrants is \((1 - u)(1 - G_1(w))\tilde{\lambda}_1 F_1(w)\)). In steady-state, \( G_1(w) \) is thus derived from the following equilibrium flows condition:

\[
\begin{align*}
(u\lambda_0 F_1(w) + (1 - u)(1 - G_1(w))\tilde{\lambda}_1 F_1(w)) &= (1 - u) \left[ G_1(w) \left[ \delta + \tilde{\lambda}_1(1 - F_1(w)) \right] \right] \\
&+ [1 - F(w)] \int_w^w \lambda_1(s(y)) dG_1(y) + \int_w^w \lambda_2(s(y)) dG_1(y)
\end{align*}
\]

\((3.1)\)

We assume that employees in high-productivity jobs segment could only be recruited from low-productivity jobs segment. They stop searching on-the-job, face no more firing risk and retire from the market at rate \( \mu \). Therefore, the steady-state c.d.f. of wage earnings, denoted as \( G_2 \), is derived from the following equilibrium flow condition:

\[
n_2 G_2(b(w)) \mu = (1 - u) \int_w^w \lambda_2(s(y)) dG_1(y)
\]

\((3.2)\)

where \( b(w) \) represents the negotiated wage of a worker who previously earned \( w \) in a low productivity job, and \( n_2 \) is the number of employees in high productivity jobs which, by the boundary condition \( G_2(b(\overline{w})) = 1 \), solves:

\[
n_2 = \frac{\lambda_0}{(\lambda_0 + \delta) \mu} \int_w^{\overline{w}} \lambda_2(s(y)) dG_1(y)
\]

\((3.3)\)
3.2.2 Value functions and reservation wage determination

Let $U$, $W_1$ and $W_2$ denote the expected discounted lifetime income of unemployed workers, employees in low and high productivity jobs, respectively. Unemployed workers receive unemployment benefits $z$ and search for job offer. A binding minimum wage $w$ is assumed to make all wage offers acceptable from the perspective of the unemployed. Similarly, workers employed at wage $w$ in low productivity jobs expect in the future either a wage increase or a promotion, but also face the reallocation shock and the firing risk. Lastly, workers in high productivity jobs receive the negotiated wage $b$ and retire from the market at rate $\mu$. So formally, these three value functions are respectively written as:

\[
\begin{align*}
    r\mathcal{U} &= z + \lambda_0 \int_{w}^{\infty} [W_1(y) - \mathcal{U}] dF_1(y) \\
    rW_1(w) &= \max_{s \geq 0} \left\{ f(w, s) + \delta (\mathcal{U} - W_1(w)) + \lambda_1(s) \int_{w}^{\infty} [W_1(y) - W_1(w)] dF_1(y) + \tilde{\lambda}_1 \int_{w}^{\infty} [W_1(y) - W_1(w)] dF_1(y) \right\} \\
    rW_2(b) &= b - \mu W_2(b)
\end{align*}
\]

where $r$ denotes the interest rate. The utility function $f(w, s)$ satisfies the following properties: $f_w(w, s) > 0$, $f_s(w, s) < 0$, $f_{ww}(w, s) \leq 0$ and $f_{ss}(w, s) \leq 0$.

The reservation wage of a worker employed at wage $w$ in a low productivity job, denoted as $x(w)$, is the wage that makes the employee indifferent between accepting a high productivity job offer and staying in the current job. Solving $W_1(w) = W_2(x(w))$ gives:

\[
\frac{r}{r + \mu} x(w) = f(w, s) + \delta \left[ \mathcal{U} - \frac{x(w)}{r + \mu} \right] + \lambda_1(s) \int_{w}^{\infty} [W_1(y) - W_1(w)] dF_1(y) + \tilde{\lambda}_1 \int_{w}^{\infty} [W_1(y) - W_1(w)] dF_1(y)
\]
3.2. MODEL

\[ x(w) = \left( \frac{r + \mu}{r + \delta} \right) \left\{ f(w, s) + \delta U + \lambda_1(s) \int_w^w [W_1(y) - W_2(x(w))] dF_1(y) \right. \]
\[ + \tilde{\lambda}_1 \int_w^w [W_1(y) - W_2(x(w))] dF_1(y) \} \]

The reservation wage \( x(w) \) is positively related to the value of unemployment \( U \) and the transition rate \( \lambda_1(s) \). Differentiating the above equation with respect to \( w \) implies:

\[ x'(w) = \left( \frac{r + \mu}{r + \delta} \right) \left\{ f_w(w, s) - x'(w)W'_2(x(w)) \left\{ \lambda_1(s)[1 - F_1(w)] + \tilde{\lambda}_1 \right\} \right\} \]

Substituting \( W'_2(x(w)) = 1/(r + \mu) \) into this equation yields:

\[ x'(w) = f_w(w, s) \left( \frac{r + \mu}{r + \delta + \lambda_1(s)[1 - F_1(w)] + \tilde{\lambda}_1} \right) > 0 \] (3.4)

The reservation wage \( x(w) \) is thus unambiguously increasing with \( w \).

3.2.3 Wage setting

3.2.3.1 Wage posting in low-productivity jobs

Unemployed workers accept all wage offers above the minimum wage, whereas employees in low productivity jobs accept offers only if these offers exceed their current wage. The unconditional probability that an offer \( w \) will be accepted by a randomly contacted worker, represented by \( h_1(w) \), is:

\[ h_1(w) = \frac{\lambda_0u + (1 - u) \int_w^w \lambda_1(s(y))dG_1(y) + (1 - u)\tilde{\lambda}_1}{\lambda_0u + (1 - u) \int_w^w \lambda_1(s(y))dG_1(y) + (1 - u)\tilde{\lambda}_1} \]
\[ = \frac{\delta + \int_w^w \lambda_1(s(y))dG_1(y) + \tilde{\lambda}_1}{\delta + \int_w^w \lambda_1(s(y))dG_1(y) + \tilde{\lambda}_1} \]

On the other side, a match separation occurs because of a job destruction (at rate \( \delta \)), because of an external mobility towards other low productivity jobs (at rate \( \lambda_1(s(w))[1 - F_1(w)] \)), because of a promotion towards high productivity jobs (at rate \( \lambda_2(s(w)) \)), or because of a job reallocation shock (at rate \( \tilde{\lambda}_1 \)). Hence, the employer’s value of a continuing match, \( J_1(w) \), solves the asset pricing equation:

\[ rJ_1(w) = p_1 - w - \left[ \delta + \tilde{\lambda}_1 + [1 - F_1(w)]\lambda_1(s(w)) + \lambda_2(s(w)) \right] J_1(w) \]
Putting it differently,

\[ J_1(w) = \frac{p_1 - w}{r + d_1(w)} \]

\[ d_1(w) = \delta + \tilde{\lambda}_1 + [1 - F_1(w)]\lambda_1(s(w)) + \lambda_2(s(w)) \]

where \( d_1(w) \) denotes the job separation rate.

The original point here is that \( d_1(w) \) will depend on the way how search effort reacts to opportunities of acceding to high productivity jobs. Importantly, there is still an incentive for the highest-paid workers in the lower occupational category to search on-the-job for promotions.

In low-productivity jobs, the wage is posted by the firm. An employer’s expected profit flow per worker contacted is the product of the hire probability per worker contacted and the value of filling a job vacancy. Formally, the wage posting policy of the firm is written as:

\[ w = \arg\{\max_{w \geq w} h_1(w)J_1(w)\} \]

The distribution of wage offers in low productivity jobs, \( F_1(w) \), is then characterized by the following first order condition:

\[ \frac{h_1'(w)}{h_1(w)} = \frac{1}{p_1 - w} + \frac{d_1'(w)}{r + d_1(w)} \]

(3.5)

3.2.3.2 Wage bargaining in high-productivity jobs

We assume that employees in high-productivity jobs segment stop searching on-the-job, face no more firing risk and retire from the market at rate \( \mu \). The value of a high productivity job, \( J_2(b) \), then solves the following asset pricing equation

\[ rJ_2(b) = p_2 - b - \mu J_2(b) \]

In high-productivity jobs, the wage rate is negotiated by the firm and the worker after they meet. This view of dual wage determination rules is supported by several empirical studies using French data. Cahuc, Postel-Vinay and Robin[2006] emphasize that low-skilled workers are mainly related to on-the-job search and wage posting, whereas the impact of bargaining power is much more important for high-skilled workers.
Let $\xi$ represent the bargaining power of the worker, the bargaining game defined by $\max_b[W_2(b) - W_1(w)]^{1-\xi} J_2(b)^{\xi}$ yields the following sharing rule:

\[
W_2(b(w)) - W_1(w) = \xi \left[ J_2(b(w)) + W_2(b(w)) - W_1(w) \right] \\
= \left( \frac{\xi}{1-\xi} \right) J_2(b(w)) \\
= \left( \frac{\xi}{1-\xi} \right) \frac{p_2 - b(w)}{r + \mu}
\]

so that $b = b(w)$. It is then straightforward to see that the bargaining gain (surplus) of the worker diminishes with his wage in the low productivity job. Differentiating the above sharing rule with respect to $w$ yields:

\[
b'(w) = \frac{W'_1(w)}{W'_2(b(w)) + \left( \frac{\xi}{1-\xi} \right) \frac{1}{r+\mu}} > 0
\]

Naturally, the negotiated wage $b(w)$ is increasing with wage earnings of the employee in a low productivity job.

By definition of the reservation wage, $W_1(w) = W_2(x(w))$, the sharing rule can also be rewritten as:

\[
W_2(b(w)) - W_2(x(w)) = \left( \frac{\xi}{1-\xi} \right) \frac{p_2 - b_2(w)}{r + \mu} \\
\frac{b(w) - x(w)}{r + \mu} = \left( \frac{\xi}{1-\xi} \right) \frac{p_2 - b(w)}{r + \mu}
\]

Finally, we have:

\[
b(w) = \xi p_2 + (1-\xi) x(w) \quad (3.6)
\]

This is a rather standard wage equation. The negotiated wage is indeed a weighted average of the productivity of a high productivity job and the reservation wage of the worker.

### 3.2.4 Search effort and job-to-job transitions

#### 3.2.4.1 Optimal search effort decision

By maximizing the value function $W_1(w)$, an employee in a low productivity job chooses his optimal search effort. We assume that the offer arrival rates,
\( \lambda_1(s) \) and \( \lambda_2(s) \), are linear functions of search effort:

\[
\lambda_i(s) = \hat{\lambda}_i + \lambda_i s, \quad \forall i = 1, 2
\]

(3.7)

where \( \hat{\lambda}_i, \lambda_i \geq 0, \ i = 1, 2 \). The optimal search effort of the worker must satisfy the following first-order condition:

\[
-f_s(w, s) = \lambda_1 \int_w^\bar{w} [W_1(y) - W_1(w)] dF_1(y) + \lambda_2 [W_2(b(w)) - W_1(w)]
\]

so that \( s = s(w) \). Differentiating this optimal condition once again with respect to \( w \) yields:

\[
-f_{ss}(w, s(w))s'(w) = f_{sw}(w, s(w)) - \lambda_1 [1 - F_1(w)] W'_1(w) + \lambda_2 [W'_2(b(w)) b'(w) - W'_1(w)]
\]

Note that the wage bargaining game implies:

\[
[W_2(b(w)) - W_1(w)]' = \left( \frac{\xi}{1 - \xi} \right) \frac{-b'(w)}{r + \mu}
\]

Therefore,

\[
-f_{ss}(w, s(w))s'(w) = f_{sw}(w, s(w)) - \lambda_1 [1 - F_1(w)] W'_1(w) - \lambda_2 \left( \frac{\xi}{1 - \xi} \right) \frac{b'(w)}{r + \mu}
\]

(3.8)

The optimal search effort of the worker is then fully characterized by the above differential equation and the following boundary condition:

\[
-f_s(\bar{w}, s(\bar{w})) = \frac{\lambda_2 \xi [p_2 - x(\bar{w})]}{r + \mu}
\]

What is the precise role of promotion in worker’s search effort strategy?

- To discuss this point, let’s first assume \( \lambda_2 = 0 \) (no promotion). In this case we must have \( s(\bar{w}) = 0 \). Note that there exists a restriction on the sign of the cross-partial \( f_{sw}(w, s) \) to guarantee that the equilibrium search effort is well defined. While \( w = \bar{w} \), we have:

\[
-f_{ss}(\bar{w}, s(\bar{w}))s'(\bar{w}) = f_{sw}(\bar{w}, s(\bar{w})) \leq 0
\]
Note that the fact \( s(\bar{w}) = 0 \) implies \( s'(\bar{w}) \leq 0 \). As by definition \( f_{ss} \leq 0, \forall w \), we must have \( f_{sw}(\bar{w}, s(\bar{w})) \leq 0 \). Consequently, \( f_{sw}(w, s(w)) \leq 0, \forall w \). Next, applying this restriction in equation (3.8), it then appears that \( s'(w) \leq 0, \forall w \). A similar result is also obtained by Christensen et al.[2002]. Since the probability to meet a wage offer higher than worker’s current earning diminishes with \( w \), the marginal value of searching must decrease with current wage and ultimately falls to zero for the highest paid workers in low productivity jobs. Accordingly, the marginal cost of search must be separable or increasing in wages, that is, \( f_{sw}(w, s) \leq 0 \).

- In turn, if \( \lambda_2 > 0, s(\bar{w}) > 0 \): it is still of interest for the highest paid workers to search on-the-job, since they may expect to accede to high-productivity jobs. Contrarily to Christensen et al.[2002], the possibility of promotion allows to avoid the boundary condition \( s(\bar{w}) = 0 \). In this case, the restriction on the sign of \( f_{sw}(w, s) \) no longer exists, the shape of search effort then depends on the sign of this cross-partial:

- if \( f_{sw}(w, s) \leq 0 \), all forces push search effort to decrease with wages. A higher wage indeed implies: (i) less possibility to obtain a wage increase in low productivity jobs, (ii) lower surplus from wage bargaining with a high productivity firm, and (iii) higher marginal cost of search.

- if \( f_{sw}(w, s) > 0 \), a higher wage reduces the marginal cost of search. If this effect is large enough, it can induce an increasing relationship between search effort and wage earnings. When \( f_{sw}(w, s) > 0 \), consumption and leisure are actually substitutable goods (in sense of Edgeworth). This property allows to capture the fact that lower-paid workers who already support a loss of consumption compensate this penalty by more leisure, hence less search effort, than higher-paid workers. Such result typically occurs with non-separable CRRA preferences between consumption and leisure and a relative risk aversion parameter greater than one.
3.2.4.2 Job-to-job transitions

We decompose total flows of job-to-job transition into two independent components: ascending mobility (promotion) and external mobility (without promotion). Let \( m_p(w) \) denote the probability of transition from a low productivity job to a high productivity job (promotion), \( m_e(w) \) the probability of external mobility within the same jobs segment, and \( m_t(w) \) the total job-to-job transition rate. Formally, these three types of mobility are defined as:

\[
\begin{align*}
    m_p(w) &= \lambda_2(s(w)) \\
    m_e(w) &= [1 - F_1(w)]\lambda_1(s(w)) + \tilde{\lambda}_1 \\
    m_t(w) &= m_p(w) + m_e(w)
\end{align*}
\]

Assuming that \( \lambda_1(s) \) and \( \lambda_2(s) \) are linear functions of \( s \) (as defined by (3.7)), the impact of wage earnings on these transition rates are given by:

\[
\begin{align*}
    m'_p(w) &= \lambda_2 s'(w) \\
    m'_e(w) &= \lambda_1 s'[w[1 - F_1(w)] - F'_1(w)\lambda_1(s(w))] \\
    m'_t(w) &= m'_p(w) + m'_e(w)
\end{align*}
\]

It is thus obvious that if \( \lambda_2 = 0 \), the probability of job-to-job transition is always decreasing with wage, i.e., \( m'_t(w) \leq 0, \forall w \), since \( s'(w) \leq 0 \) and \( F'_1(w) \geq 0 \). In contrast, if \( s'(w) > 0 \) at least for some higher wages, the relation between the probability of job-to-job transition and the wage could be ambiguous.

3.2.5 Labor market equilibrium

The appendix A provides a detailed derivation of the labor market equilibrium. Labor market equilibrium in low-productivity jobs segment can actually be summarized by a three-dimensional differential system which jointly defines \( \{F_1(w), s(w), x(x)\} \):

\[
F'_1(w) = \frac{\delta + \lambda_1 + [1 - F_1(w)]\lambda_1(s(w)) + \lambda_2(s(w))}{2\lambda_1(s(w))[p_1 - w]} + \frac{s'(w)[\lambda_2 + \lambda_1(1 - F_1(w))]}{2\lambda_1(s(w))}
\]

\footnote{For \( r- > 0 \), the first equation collapse to:}

\[
F'_1(w) = \frac{\delta + \lambda_1 + [1 - F_1(w)]\lambda_1(s(w)) + \lambda_2(s(w))}{2\lambda_1(s(w))[p_1 - w]} + \frac{s'(w)[\lambda_2 + \lambda_1(1 - F_1(w))]}{2\lambda_1(s(w))}
\]
3.2. MODEL

\[ F'_1(w) = \frac{1}{m_1 - w} + \frac{s'(w)[\lambda_2 + \lambda_1(1 - F_1(w))]}{\lambda_1(s(w)) + \lambda_3(s(w))[1 - F_1(w)] + \lambda_2(s(w))} \]

\[ s'(w) = \frac{f_{sw}(w, s) - \lambda_1[1 - F_1(w)] \left( \frac{f_{w(s, s)}(w) - \lambda_2(s(s)) \xi'(w)}{r + \delta + \lambda_1(s(s))[1 - F_1(w)]} \right) - \frac{\lambda_2(s(s))}{r + \mu} \xi x'(w)}{-f_{ss}(w, s)} \]

\[ x'(w) = \frac{(r + \mu)f_{w}(w, s)}{r + \delta + \lambda_1(s(w))[1 - F_1(w)] + \lambda_1} \]

where \( F_1(w), s(w) \) and \( x(w) \) satisfy the three boundary conditions:

\[ F_1(w) = 0 \]

\[ -f_s(w, s) = \frac{\lambda_3 \xi [p_2 - x(w)]}{r + \mu} \]

\[ x(w) = \frac{(r + \mu)(r + \lambda_0)}{r(r + \delta + \lambda_0)} \left\{ f(w, s(w)) + \frac{\delta z}{r + \lambda_0} + \left( \lambda_1(s(w)) + \tilde{\lambda}_1 + \frac{\delta \lambda_0}{r + \lambda_0} \int_\omega \frac{1 - F_1(y)}{r + \delta + \tilde{\lambda}_1 + [1 - F_1(y)] \lambda_1(s(y))} dy \right) \right\} \]

Given \( F_1(w), s(w) \) and \( x(w) \), it is then straightforward to determine the distributions of wage earnings in low-productivity and high-productivity jobs segments:

\[ G'_1(w) = F'(w) \left[ \frac{\delta + \tilde{\lambda}_1 + \int_\omega \lambda_1(s(y)) dG_1(y)}{\delta + \tilde{\lambda}_1 + [1 - F_1(w)] \lambda_1(s(y)) + \lambda_2(s(y))} \right] \]

\[ G'_2(b(w)) = \left( \frac{\lambda_0}{\lambda_0 + \delta n_2 e} \right) \left( \frac{\lambda_2(s(w)) G'_1(w)}{(1 - \xi)x'(w)} \right) \]

which satisfy the boundary conditions:

\[ G_1(w) = 0 \]

\[ G_2(b(w)) = 0 \]

where \( n_2 \) solves equation (3.3) and \( b(w) \) is defined by equation (3.6).
3.3 Computational experiments

3.3.1 Calibration

A parametric specification of the utility function \( f(w, s) \) is required. In order to allow the marginal cost of search to be either increasing or decreasing with wages, we consider the following CRRA specification:

\[
f(w, s) = \begin{cases} 
\frac{(w^{(1-h-s)}(1-\alpha))^{1-\rho}}{1-\rho} & \text{(if } \rho \neq 1) \\
\alpha \log(w) + (1-\alpha) \log(1-h-s) & \text{(if } \rho = 1) 
\end{cases}
\]

with \( \rho \geq 0, \alpha \in [0, 1] \), so that we have \( f_w > 0, f_s < 0, f_{ww} < 0, f_{ss} < 0 \). The sign of the cross-partial \( f_{sw} \) then depends on \( \rho \): if \( \rho \leq 1 \), \( f_{sw} \leq 0 \) whereas if \( \rho > 1 \), \( f_{sw} > 0 \).

This model is calibrated annually in order to reflect French economy, and more specifically the “intermediate professions” occupation. Two sets of parameters, dissociated by the calibration procedure, are distinguished. The first set, marked with \( \Phi_1 = \{r, h, w, z, \mu, \alpha, \delta, \lambda_0, \tilde{\lambda}_1, \rho\} \) where \( \dim(\Phi_1) = 11 \), is calibrated based on external information. First, the real interest rate is \( r = 4\% \) per annum. Further, we set in a fairly standard way \( h = 1/3 \) and \( \alpha = 2/3 \). The minimum wage rate \( w \) and the unemployment compensation \( z \) are jointly fixed: minimum wage is normalized to be unity, which corresponds to 1.3 times of average unemployment compensation received by the unemployed workers. Four parameters are calibrated based on statistical information deduced from French Labor Survey (FLS 1990-1999). The annual job destruction rate, \( \delta \), is fixed at 2\%. The offer arrival rate when unemployed, \( \lambda_0 \), accounts for the fact that the unemployment spell for the technicians is on average 17 months. The probability of a job reallocation shock is the yearly frequency that the highest-paid workers experience simultaneously a change of firm and a wage decrease\(^2\), that is, \( \tilde{\lambda}_1 = 3\% \). The probability of retirement from high-productivity jobs segment is lastly calibrated so as to account for the fact that the technicians who experience a transition to the

\(^2\)By “the highest-paid workers”, we mean the workers employed at wages above the 99th percentile in the SPC “intermediate professions”.
manager occupation are on average 36 years old, that is, they will work 24 years in the manager occupation before retirement. Accordingly, the annual probability of retirement is set to \( \mu = 4.2\% \). Existing empirical studies suggest that bargaining power of workers is significant but low. In line with Cahuc, Postel-Vinay and Robin[2006], we set \( \xi = 13\% \) which is an intermediate value over industries. The parameter of risk-aversion is fixed at \( \rho = 3 \). All these parameter values are reported in table 3.2.

A second set of parameters, marked with \( \Phi_2 = \{p_1, p_2, \hat{\lambda}_1, \lambda_1, \hat{\lambda}_2, \lambda_2\} \) where \( \dim(\Phi_2) = 6 \), is calibrated to reproduce some stylized facts on wage dispersion and workers mobilities. First, the productivity values \( p_1 \) and \( p_2 \) are calibrated to reproduce the wage dispersion characteristics for Technicians: (1) the ratio of the mean wage on the minimum wage is 1.85, (2) the ratio of the 5th wage decile on the 1st decile \( (D_5/D_1) \) equals 1.46. Four parameters that control the job contact rates, \( \hat{\lambda}_1, \lambda_1, \hat{\lambda}_2 \) and \( \lambda_2 \), are jointly determined to reproduce (1) an average yearly promotion rate of 3.7\%, (2) an average yearly transition rate of 7.6\%, (3) the average external mobility rate for workers whose wages lie in the 5th decile equals 3.9\%, (4) the average transition rate for workers whose wages lie in the 5th decile equals 7.1\%. These parameters are reported in the table 3.3.

<table>
<thead>
<tr>
<th>Table 3.2: Exogenous Parameter Set ( \Phi_1 )</th>
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<tbody>
<tr>
<td>( r )</td>
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<tr>
<td>0.04</td>
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<table>
<thead>
<tr>
<th>Table 3.3: Endogenous Parameter Set ( \Phi_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_1 )</td>
</tr>
<tr>
<td>3.5</td>
</tr>
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</table>
Figure 3.1: Optimal Search Effort
3.3.2 Simulation results and discussion

The figure 3.1 plots the optimal search effort decision of the worker. From the figure, the shape of search effort turns to be first decreasing then increasing with the wage. Importantly, we have \( s(\overline{w}) > 0 \). These results clearly confirm that if consumption and leisure are substitutable (sign of cross partial \( f_{sw}(w, s) > 0 \), i.e., parameter of risk aversion \( \rho > 1 \)), the higher-paid workers in low productivity jobs will compensate the loss due to less chance of wage increase in the same jobs segment by searching more for acceding to high productivity jobs. By this way, equilibrium wage distribution in either jobs segment need not to be monotonically increasing with wage, more realistic shape of wage distribution is then allowed by the model.

The figure 3.2 plots the predicted wage distributions in both jobs segments. It is clear that the log-normal shape of wage density in low-productivity jobs segment, \( g_1(w) \), is particularly well replicated by the model. We then argue that if the opportunities of promotion are incorporated into the equilibrium search model, a bell-shaped wage density will be a typical equilibrium outcome. This argument is true even though all workers are identical ex ante. In most equilibrium search models (including this one), a high wage reduces the probability of external mobility because the probability of wage increase decreases with the wage rate. This induces a positive relation between the duration of employment and the wage. However, the employee will compensate this loss by searching more for acceding to high productivity jobs. This in turn induces a negative relation between the duration of employment and the wage, so represents an opposite force to the positive one. The optimal wage policy of the firm trades off these two competing effects. As we have seen, it leads to a market equilibrium where a bell-shaped wage density will be a typical equilibrium outcome.

It should be pointed out that the wage density in high-productivity jobs segment, \( g_2(b) \), is also bell-shaped. Because the inflow into the high-productivity jobs segment is the workers in low productivity jobs who search on-the-job for promotion whereas the outflow is the workers in high productivity jobs who retire from the market, wage distribution in high-productivity jobs segment
Figure 3.2: Equilibrium Wage Distribution
3.3. COMPUTATIONAL EXPERIMENTS

Figure 3.3: Job-to-job Mobilities

Promotion Rate

External Mobility Rate

Transition Rate

Mobility
Mobility With Wage Increase
is a transformation of wage distribution in low-productivity jobs segment. Therefore, a bell-shaped wage density in low-productivity jobs segment will typically induce a bell-shaped wage density in high-productivity jobs segment\(^3\). Consequently, the model allows that wage distribution within either jobs segment is bell-shaped.

As the probability of job-to-job transition could be decomposed into probability of promotion and probability of external mobility, the figure 3.3 plots the predicted probability of these three types of mobility respectively with the wage. For workers employed at wages above the 5th decile, their probability of being promoted is increasing with the wage. On the other side, the probability of job-to-job transition is strictly decreasing with wage. These results indicate that the \(U\)-shaped relation between frequency of job-to-job transition and wage that we observe in the French data could not be entirely due to endogenous search effort of the worker.

All the results up to the point are based on the assumption that consumption and leisure are actually substitutable, i.e., \(f_{sw} > 0 (\rho > 1)\). As shown in the theoretical section, the sign of the cross-partial \(f_{sw}\) (the value of \(\rho\)) is the key determinant of the profile of search effort. If the cross-partial \(f_{sw} \leq 0 (\rho \leq 1)\), a higher wage indeed implies: (i) less possibility to obtain a wage increase in low productivity jobs, (ii) lower surplus from wage bargaining with a high productivity firm, and (iii) higher marginal cost of search. All forces push search effort to decrease with wage. The figure 3.4 illustrates this point for the case \(\rho = 1\). Note, particularly, that we have \(s(\bar{w}) > 0\) all the same even if \(\rho = 1\). The underlying reason is that highest-paid workers could also search for promotion because \(\lambda_2 > 0\). In this case, we return to the Burdett-Mortensen\}[1998\] framework where the densities of wage offer and wage earnings in low-productivity jobs segment are both increasing functions of wage. In consequence, wage density in high-productivity jobs

\(^3\)We could find that the shape of the density of negotiated wage, \(g_2(b)\), is not very beautiful. Several extensions of the model could help this predicted density to be close to the actually observed one. For example, if allow for the possibility that the employees of the high-productivity jobs segment could also be recruited from unemployment, the peak of the density of negotiated wage will move towards the left.
3.3. COMPUTATIONAL EXPERIMENTS

Figure 3.4: Impact of risk-aversion on market equilibrium ($\rho = 1$)
segment, \( g_2(b) \), is also strictly increasing with the negotiated wage \( b \).

### 3.4 Conclusion

In the literature, the flow of job-to-job transition from one occupational category (SPC) to another is a dimension up to now largely neglected. We argue that extending the equilibrium search model to account for this dimension is a fruitful way to revisit the shape of equilibrium wage distribution. In this chapter, we reinspect the role of endogenous search effort in a pure environment where all workers and firms are respectively identical. We distinguish two types of job. Workers in low productivity jobs are allowed to search on-the-job not only for wage increases, but also for opportunities of promotion. Once a worker accedes to a high productivity job, it is assumed that the wage is negotiated according to a sharing rule. We also assume that employees in high-productivity jobs segment could only be recruited from low-productivity jobs segment, they stop searching on-the-job and face no more firing risk. We show that, depending on the non-separability of preferences between consumption and leisure, the model allows to capture that wage distribution within either jobs segment is bell-shaped.

This chapter brings to two basic conclusions. First, worker search effort could be a force that contributes to explain the essence of actual wage inequality and wage dispersion. The underlying reason is that worker search effort directly affects the probability of job-to-job mobility. Next, worker search effort is, by itself, not sufficient to explain the essence of the relation between frequency of promotion and wage, as well as the relation between frequency of job-to-job transition and wage, that we observed in the data. This result leads to a further investigation of the impact of firm behavior to job-to-job transition and, in particular, worker promotion.
Appendix A: Explicit Equilibrium Deduction

The labor market equilibrium is characterized by

\[ \Upsilon = \{ F_1(w), G_1(w), s(w), x(w), b(w), G_2(w), n_2 \} \]

with \( \dim(\Upsilon) = 7 \). The system is defined on the support \( w_1 \in [\underline{w}_1, \bar{w}_1] \), and \( w_2 = b(w_1) \). The sub-system \( \{ F_1(w), s(w), x(w), G(w) \} \) is calculated by way of ODE (ordinary differential equation) algorithm. Note that the first three differential equations are independent of \( G(w) \). Known \( \{ F_1(w), s(w), x(w), G(w) \} \), it is straightforward to solve other three functions.

First we write the ODE system:

- \( F'_1(w) \)

  From

  \[
  \frac{1}{p_1 - w} = \frac{[\hat{\lambda}_1 + \lambda_1 s(w)]G'_1(w)}{\delta + \int_{\underline{w}_1}^{w} [\hat{\lambda}_1 + \lambda_1 s(y)]dG_1(y) + \hat{\lambda}_1} + \frac{F'_1(w)[\hat{\lambda}_1 + \lambda_1 s(w)] - s'(w)\{\lambda_2 + \lambda_1 (1 - F_1(w))\}}{r + \delta + \hat{\lambda}_1 + [1 - F_1(w)][\hat{\lambda}_1 + \lambda_1 s(w)] + [\hat{\lambda}_2 + \lambda_2 s(w)]}
  \]

  and

  \[
  G'_1(w) = F'(w) \left[ \frac{\delta + \hat{\lambda}_1 + \int_{\underline{w}_1}^{w} [\hat{\lambda}_1 + \lambda_1 s(y)]dG_1(y)}{\delta + \hat{\lambda}_1 + [1 - F_1(w)][\hat{\lambda}_1 + \lambda_1 s(w)] + [\hat{\lambda}_2 + \lambda_2 s(w)]} \right]
  \]

  we will have

  \[
  F'_1(w) = \frac{\frac{1}{p_1 - w} + \frac{s'(w)[\lambda_2 + \lambda_1 (1 - F_1(w))]}{r + \delta + \hat{\lambda}_1 + [1 - F_1(w)][\hat{\lambda}_1 + \lambda_1 s(w)] + [\hat{\lambda}_2 + \lambda_2 s(w)]}}{\delta + \lambda_1 + [1 - F_1(w)][\hat{\lambda}_1 + \lambda_1 s(w)] + [\lambda_2 + \lambda_2 s(w)] + \frac{[\hat{\lambda}_1 + \lambda_1 s(w)]}{r + \delta + \hat{\lambda}_1 + [1 - F_1(w)][\hat{\lambda}_1 + \lambda_1 s(w)] + [\hat{\lambda}_2 + \lambda_2 s(w)]}}
  \]

  with the boundary condition

  \[ F_1(w) = 0 \]
\( s'(w) \)

From

\[
-f_{ss}(w, s)s'(w) = f_{sw}(w, s) - \lambda_1 [1 - F_1(w)] W'_1(w) - \frac{\lambda_2 \xi x'(w)}{r + \mu}
\]

and

\[
W'_1(w_1) = \frac{f_w(w, s) + [\hat{\lambda}_2 + \lambda_2 s] b'(w)}{r + \delta + \tilde{\lambda}_1 + [1 - F_1(w)][\lambda_1 + \lambda_1 s] + [\hat{\lambda}_2 + \lambda_2 s]}
\]

we will have

\[
s'(w) = \frac{f_{sw}(w, s) - \lambda_1 (1 - F_1(w)) \left( \frac{f_{sw}(w, s) + [\hat{\lambda}_2 + \lambda_2 s] \frac{1 - \xi x'(w)}{r + \mu}}{r + \delta + \tilde{\lambda}_1 + [1 - F_1(w)][\lambda_1 + \lambda_1 s] + [\hat{\lambda}_2 + \lambda_2 s]} \right) - \frac{\lambda_2 \xi x'(w)}{r + \mu}}{-f_{ss}(w, s)}
\]

so that

\[
s'(w) = \frac{1}{(\alpha - 1)[(1 - \alpha)(1 - \rho) - 1] w^{\alpha(1 - \rho)}(1 - h - s)^{(1 - \alpha)(1 - \rho) - 1} \times}
\]

\[
\left\{ (\alpha - 1)\alpha (1 - \rho) w^{\alpha(1 - \rho) - 1} (1 - h - s)^{(1 - \alpha)(1 - \rho) - 1} - \frac{\lambda_2 \xi x'(w)}{r + \mu}
\right.
\]

\[
-\lambda_1 [1 - F_1(w)] \left[ \frac{[\alpha w^{\alpha(1 - \rho) - 1} (1 - h - s)^{(1 - \alpha)(1 - \rho)}] + [\hat{\lambda}_2 + \lambda_2 s(w') \xi x'(w)]}{r + \delta + \tilde{\lambda}_1 + [1 - F_1(w)][\lambda_1 + \lambda_1 s(w)] + [\hat{\lambda}_2 + \lambda_2 s(w)]} \right]
\]

(A-2)

with the boundary condition

\[
s(w) = 1 - h - \left[ \frac{\lambda_2 \xi}{r + \mu} \right] \left( \frac{p_2 - x(w)}{(1 - \alpha) w^{\alpha(1 - \rho)}} \right) (1 - \alpha)^{(1 - \rho) - 1}
\]

where

\[
INT = \int_w^\infty [1 - F_1(y)] W'_1(y) dy
\]

\[
= \int_w^\infty \left[ 1 - F_1(y) \right] \left\{ \frac{[\alpha y^{\alpha(1 - \rho) - 1} (1 - h - s(y))^{(1 - \alpha)(1 - \rho)}] + [\hat{\lambda}_2 + \lambda_2 s(y) \xi x'(y)]}{r + \delta + \tilde{\lambda}_1 + [1 - F_1(y)][\lambda_1 + \lambda_1 s(y)] + [\hat{\lambda}_2 + \lambda_2 s(y)]} \right\} dy
\]

(A-3)
• \( x'(w) \)

From the definition \( W_1(w_1) = W_2(x(w)) \), it is easy to show that

\[
x'(w) = \frac{(r + \mu) \left[ \alpha w^{\alpha (1 - \rho) - \lambda_1} (1 - s(w)) \right]^{(1 - \alpha)(1 - \rho)}}{r + \delta + [\lambda_1 + \lambda_1 s(w)][1 - F_1(w)] + \lambda_1} \tag{A-4}
\]

with the boundary condition

\[
x(w) = \left( r + \delta + \xi [\lambda_2 + \lambda_2 s(w)] \right) - \frac{\delta \lambda_0}{(r + \lambda_0)(r + \mu)} - 1 \times \left\{ \left( \frac{w^{\alpha (1 - h - s(w)) (1 - \rho)}}{1 - \rho} \right) + \frac{\delta}{r + \lambda_0} \left( \frac{z^{\alpha - \rho}}{1 - \rho} \right) + \frac{\delta \lambda_0}{r + \lambda_0} \times INT \right\}
\]

• \( G'_1(w) \)

We know

\[
\delta F_1(w) + (1 - G_1(w)) \tilde{\lambda}_1 F_1(w) = \left\{ G_1(w) \left[ \delta + \tilde{\lambda}_1 (1 - F_1(w)) \right] + [1 - F(w)] \int_{w_1}^{w} [\lambda_2 + \lambda_1 s(y)] dG_1(y) + \int_{w_1}^{w} [\tilde{\lambda}_2 + \lambda_2 s(y)] dG_1(y) \right\}
\]

Since

\[
\int_{w_1}^{w} [\lambda_2 + \lambda_2 s(y)] dG_1(y) = \left( \lambda_2 - \frac{\lambda_2 \tilde{\lambda}_1}{\lambda_1} \right) G_1(w) + \left( \frac{\lambda_2}{\lambda_1} \right) \int_{w_1}^{w} [\lambda_1 + \lambda_1 s(y)] dG_1(w)
\]

we will have

\[
\int_{w_1}^{w} [\tilde{\lambda}_1 + \lambda_1 s(y)] dG_1(y) = \frac{F_1(w) \left[ \delta + \tilde{\lambda}_1 \right] - G_1(w) \left[ \delta + \tilde{\lambda}_1 + \frac{\lambda_1 \lambda_2 - \lambda_2 \lambda_1}{\lambda_1} \right]}{[1 - F(w) + \frac{\lambda_2}{\lambda_1}]}
\]
Finally,

\[
G'_1(w) = F'(w) \left\{ \begin{array}{l}
\delta + \tilde{\lambda}_1 + \left[ \frac{F_1(w)[\delta + \tilde{\lambda}_1] - G_1(w)[\delta + \tilde{\lambda}_1 + \frac{\lambda_1 \hat{\lambda}_2 - \lambda_2 \hat{\lambda}_1}{\lambda_1}]}{[1 - F(w) + \frac{\lambda_2}{\lambda_1}]} \right] \\
\delta + \tilde{\lambda}_1 + [1 - F_1(w)][\tilde{\lambda}_1 + \lambda_1 s(w)] + [\tilde{\lambda}_2 + \lambda_2 s(w)]
\end{array} \right\}
\]

(A-5)

with the boundary condition

\[
G_1(w) = 0
\]

□ □ □

The explicit expressions of other three functions are as follows:

\[
n_2 = \frac{\lambda_0}{(\lambda_0 + \delta)\mu} \int_{\underline{w}}^{\overline{w}} [\tilde{\lambda}_2 + \lambda_2 s(y)] dG_1(y) \quad \text{(A-6)}
\]

\[
b(w) = \xi \rho_2 + (1 - \xi) x(w) \quad \text{(A-7)}
\]

\[
G'_2(b(w)) = \left( \frac{\lambda_0}{(\lambda_0 + \delta) n_2 \mu} \right) \left( \frac{[\tilde{\lambda}_2 + \lambda_2 s(w)] G'_1(w)}{(1 - \xi) x'(w)} \right) \quad \text{(A-8)}
\]
Chapter 4

Impact of firm behavior to equilibrium wage dispersion and job-to-job transition

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4.1 Introduction

The empirical investigations presented in chapter 2 indicate that the actual relation between frequency of job-to-job transition and wage is ambiguous, typically first decreasing then increasing. Putting it differently, high-wage workers are more likely to switch to a new job than low- and middle-wage workers (see figure 2.7 in chapter 2). Further, we decompose the total flows of job-to-job transition into flows of promotion (raise of occupational category) and flows of external mobility (change of firm). We show that the $U$-shaped profile between frequency of job-to-job transition and wage earnings refers principally to the flows of promotion (around two third).

Worker promotion is explained, in chapter 3, by endogenous search effort of the worker. Simulation experimentations show that the probability of job-to-job transition predicted by the model is strictly decreasing with the wage for all realistic combinations of parameter values. We thus conclude that worker search decision is not sufficient to give a satisfactory explanation of promotion, as well as the observed $U$-shaped relation between frequency of job-to-job transition and wage.

This chapter aims to give a joint explanation of equilibrium wage distribution and job-to-job transition. We investigate the impact of the firm behavior to market wage dispersion and, in particular, worker promotion. Firm heterogeneity in observable productivity and non-observable wage policy is well known to be the most important determinant of inter-industry and inter-firm wage differentials (see Mortensen [1998], Mortensen [2002b], Postel-Vinay and Robin [2002] and Abowd-Kramarz [2000a,b]). Nevertheless, the impact of firm heterogeneity to worker promotion is a subject until now less covered in the literature.

We thus construct a job search model with two types of jobs. We choose the job search model as the workhorse as it permits to deal with not only internal and external promotion (see Lazear and Oher [2004]), but also wage decline associated with job-to-job transition. We distinguish two types of jobs: low-productivity jobs and high-productivity jobs. This distinction
refers to differences in occupational categories. Let $p_1$ ($p_2$) denote the marginal productivity of a unity of efficient labor in low-productivity (high-productivity) jobs segment, respectively. We assume that $p_1$ is constant, whereas $p_2$ follows a continuous cumulative distribution function $\Gamma$, and a density function $\gamma$, defined on the support $p_2 \in [p_2, \overline{p}_2]$. Without loss of generality, we assume $p_1 < E(p_2)$. This view coincides with the idea that the more skill-intensive an occupation is, the more heterogeneous is the marginal productivity. At the bottom of the skill hierarchy, the marginal productivity of the manual workers and employees are rather homogeneous (see Postel-Vinay and Robin[2002]).

Homogenous workers start working in jobs with low productivity. They face an exogenous risk of unemployment, and search off-the-job not only for better opportunities in low productivity jobs, but also for acceding to high productivity jobs. We assume that offer arrival rates when both unemployed and employed are exogenous, but don’t depend on endogenous search effort of the worker. We also assume that employees in high-productivity jobs segment could only be recruited from low-productivity jobs segment, they stop searching on-the-job and face no more firing risk. Briefly speaking, the model describes a two-tier structure of labor market where employees in the upper segment are stable, but mobile in the lower segment. Firms are allowed to make human capital investment to low-productivity jobs. Firms also pay a fixed cost of training for each high-productivity job.

In low-productivity jobs, the wage is posted by the firm. Once a worker accedes to a high productivity job, it is assumed that the wage is negotiated according to a sharing rule. This view of dual wage determination rules is supported by several empirical studies using French data. Cahuc, Postel-Vinay and Robin[2006] emphasize that low-skilled workers are mainly related to on-the-job search and wage posting, whereas the impact of bargaining power is much more important for high-skilled workers (see table 3.1 in chapter 3).

The first characteristic of the model is that worker promotion is mainly affected by firm decision. We assume that human capital is transferable. Let $k$ represent human capital investment per worker. Let also the value of worker
product in a low (high) productivity job be an increasing concave function of the human capital, denoted as $p_1 f(k) \ (p_2 f(k))$, respectively. Firms pay a fixed cost of training for each high-productivity job. Therefore, the optimal promotion policy of the firm could be summarized by the selection of a threshold value of worker product, denoted by $\tilde{p}_2 f(k)$, such that the firm could retrieve the cost of training and exploit a positive profit from a new job. As lower-paid workers in low productivity jobs are less invested in human capital (see hereunder), once they receive an offer from high-productivity jobs segment, they are required a higher level of reservation productivity (a higher value of $\tilde{p}_2$). As the probability of being promoted decreases with the requirement of the threshold productivity, in equilibrium, it could be that lower-paid workers are less promoted whereas higher-paid workers are more likely to be promoted. By this way, the model predicts a positive relation between probability of promotion and wage and, consequently, a $U$-shaped relation between probability of job-to-job transition and wage.

The second characteristic of the model is that firms are allowed to make human capital investment to low-productivity jobs. Specially, we show that employers who offer higher wages will always invest more in human capital, even though the duration of employment is not monotonously increasing with wage. In the canonical equilibrium search models where firms are allowed to invest in human capital as in Mortensen[1998], the amount of human capital investment is positively correlated with the duration of a job because the probability of external mobility decreases with wage. In this model with two types of job, a raise of human capital also raises the probability that an employee will be promoted, which in turn reduces the expected duration of a job. In equilibrium, the relation between human capital investment and job duration is ambiguous. Consequently, human capital is always increasing with wage whereas the relation between probability of match separation and wage could well be $U$-shaped.

The third characteristic of the model is that equilibrium wage distribution is typically bell-shaped. Indeed, explanation of wage dispersion in this chapter is essentially the same as that in the last chapter. Known that effect of external mobility will induce a positive relation between the duration of employment and the wage whereas effect of promotion induces a negative
relation between the two, the optimal wage policy of the firm trades off these two competing effects and leads to a market equilibrium where a bell-shaped wage density will be a typical equilibrium outcome. On the other side, as it is assumed that jobs are stable and wages are negotiated in high-productivity jobs segment, wage distribution in high-productivity jobs segment is just a transformation of wage distribution in low-productivity jobs segment. Therefore, a bell-shaped wage density in low-productivity jobs segment will typically induce a bell-shaped wage density in high-productivity jobs segment.

This chapter is organized as follows. Section 2 describes the model. In section 3, we structurally estimate the model using the simulated method of moments (SMM), and evaluate the empirical performances of the model. The last section concludes.

4.2 Model

We thus construct a job search model with two types of jobs: low-productivity jobs and high-productivity jobs. This distinction refers to differences in occupational categories. Let $p_1$ ($p_2$) denote the marginal productivity of a unity of efficient labor in low-productivity (high-productivity) jobs segment, respectively. We assume that $p_1$ is constant, whereas $p_2$ follows a continuous cumulative distribution function $\Gamma$, and a density function $\gamma$, defined on the support $p_2 \in [\underline{p}_2, \overline{p}_2]$. Without loss of generality, we assume $p_1 < E(p_2)$. This view coincides with the idea that the more skill-intensive an occupation is, the more heterogeneous is the marginal productivity. At the bottom of the skill hierarchy, the marginal productivity of the manual workers and employees are rather homogeneous (see Postel-Vinay and Robin [2002]).

Homogenous workers start working in jobs with low productivity. They search on-the-job not only for better opportunity in low-productivity jobs segment, but also for acceding to high-productivity jobs segment. Search is sequential and non-directed. We assume that offer arrival rates when both unemployed and employed are exogenous, but don’t depend on endogenous search effort of the worker. At random time intervals, a worker receives information about a new job opening. Let $\lambda_0$ denote the offer arrival rate.
when unemployed. Similarly, let \( \lambda_1 (\lambda_2) \) denote the unity offer arrival rate of a low-productivity job (a high-productivity job), respectively.

In low-productivity jobs, the wage is posted by the firm. Once a worker accedes to a high productivity job, it is assumed that the wage is negotiated according to a sharing rule. We also assume that employees in high-productivity jobs segment could only be recruited from low-productivity jobs segment. They stop searching on-the-job, face no more firing risk and retire from the market at rate \( \mu \). In this circumstance, a transition from a low-productivity job to a high-productivity job refers to a promotion, whereas a transition from a low-productivity job to another low-productivity job refers to an external mobility (without promotion). Therefore, the model describes a two-tier structure of labor market where employees in the upper segment are stable, but mobile in the lower segment.

Firms are allowed to make human capital investment to low-productivity jobs. Human capital is assumed to be transferable from low productivity jobs to high productivity jobs. Let \( k \) represent human capital investment per worker. Let also the value of worker product be an increasing concave function of this investment denoted as \( p_1 f(k) \), where \( p_1 \) is the marginal productivity of a unity of efficient labor in a low productivity job.

Firms also pay a fixed cost of training, \( C \), for each high-productivity job. Let \( p_2 f(k) \) represent the value of worker product in a high productivity job. The optimal promotion policy of the firm could be summarized by the selection of a threshold value of worker product, denoted by \( \tilde{p}_2 f(k) \), such that the firm could retrieve the cost of training and exploit a positive profit from a new job.

### 4.2.1 Labor market flows

Suppose a labor market in steady state where all workers are identical. Because all equilibrium wage offers are acceptable, unemployed workers find employment at rate \( \lambda_0 \). Let normalize to one the sum of unemployed workers and employees in low productivity jobs. In the steady state, the balance between flows into and out of unemployment assuming a constant unemploy-
ment rate is written as:

\[(1 - u)\delta = \lambda_0 u\]

where \(\delta\) represents the exogenous job destruction rate.

Let \(G_1\) and \(F_1\) denote the cumulative distribution function of earnings and wage offers in low-productivity jobs segment, the stock of employees earning \(w\) or less in low productivity jobs is \((1 - u)G_1(w)\).

The inflow into the stock \((1 - u)G_1(w)\) is the unemployed who draws a wage offer below \(w\) (the measure of such entrants is \(u\lambda_0 F_1(w)\)). On the other side, workers leave this stock because they are laid off (which happens at rate \(\delta\)), because they receive an outside offer of a low productivity job with associated wage greater than \(w\) (at rate \(\lambda_1[1 - F(w)]\)), or because they receive an offer of a high productivity job (at rate \(\lambda_2[1 - \Gamma(\bar{p}_2)]\)). In steady-state, \(G_1(w)\) is thus derived from the following equilibrium flows condition:

\[u\lambda_0 F_1(w) = (1 - u)\left\{G_1(w)\left[\delta + \lambda_1(1 - F_1(w))\right] + \lambda_2 \int_w^w [1 - \Gamma(\bar{p}_2(y))]dG_1(y)\right\}\]

Differentiating this equation with respect to \(w\) gives

\[G'_1(w) = F'(w)\left[\frac{\delta + \lambda_1 G_1(w)}{\delta + \lambda_1(1 - F_1(w)) + \lambda_2[1 - \Gamma(\bar{p}_2(w))]}\right] \quad (4.1)\]

We assume that employees in high-productivity jobs segment could only be recruited from low-productivity jobs segment. They stop searching on-the-job, face no more firing risk and retire from the market at rate \(\mu\). Therefore, the steady-state c.d.f. of wage earnings, denoted as \(G_2\), is derived from the following equilibrium flow condition:

\[n_2 G_2(b(p_2f(k), w))\mu = (1 - u)\lambda_2 \int_w^w [1 - \Gamma(\bar{p}_2(y))]dG_1(y) \quad (4.2)\]

where \(b(p_2f(k), w)\) represents the negotiated wage of a worker who earns \(w\) in a low productivity job and draws a marginal productivity \(p_2\) once promoted, and \(n_2\) is the number of employees in high productivity jobs which, by the boundary condition \(G_2(b(w)) = 1\), solves:

\[n_2 = \frac{\lambda_0 \lambda_2}{(\lambda_0 + \delta)\mu} \int_w^w [1 - \Gamma(\bar{p}_2(y))]dG_1(y) \quad (4.3)\]
4.2. MODEL

4.2.2 Value functions and reservation wages

Let \( U, W_1 \) and \( W_2 \) denote the expected discounted lifetime income of unemployed workers, employees in low and high productivity jobs, respectively. Unemployed workers receive unemployment benefits \( z \) and search for job offer. A binding minimum wage \( w \) is assumed to make all wage offers acceptable from the perspective of the unemployed. Similarly, workers employed at wage \( w \) in low productivity jobs expect in the future either a wage increase or a promotion, but also face the firing risk. Lastly, workers in high productivity jobs receive the negotiated wage \( b \) and retire from the market at rate \( \mu \). So formally, these three value functions are respectively written as:

\[
\begin{align*}
    rU &= U(z) + \lambda_0 \int_w^\infty [W_1(y) - U] \, dF_1(y) \\
    rW_1(w) &= U(w) + \delta [U - W_1(w)] + \lambda_1 \int_w^\infty [W_1(y) - W_1(w)] \, dF_1(y) \\
        &\quad + \lambda_2 \int_{\bar{p}_2(w)}^{p_2} [W_2(b(p_2 f(k), w)) - W_1(w)] \, d\Gamma(p_2) \\
    rW_2(b(p_2 f(k), w)) &= b(p_2 f(k), w) - \mu W_2(b(p_2 f(k), w))
\end{align*}
\]

where \( r \) is the interest rate and \( U(\cdot) \) denotes the utility function which satisfies the usual properties: \( U'(\cdot) > 0, U''(\cdot) < 0 \).

The reservation wage of a worker employed at wage \( w \) in a low productivity job, denoted as \( x(w) \), is the wage that makes the employee indifferent between accepting a high productivity job offer and staying in the current job. Solving \( W_1(w) = W_2(x(w)) \) gives:

\[
\begin{align*}
    \frac{r}{r + \mu} x(w) &= U(w) + \delta \left[ U - \frac{x(w)}{r + \mu} \right] + \lambda_1 \int_w^\infty [W_1(y) - W_2(x(w))] \, dF_1(y) \\
    x(w) &= \left( \frac{r + \mu}{r + \delta} \right) \left\{ U(w) + \delta U + \lambda_1 \int_w^\infty [W_1(y) - W_2(x(w))] \, dF_1(y) \right\}
\end{align*}
\]

The reservation wage \( x(w) \) is positively related to the value of unemployment \( U \) and the transition rate \( \lambda_1 \). By differentiating the above equation with respect to \( w \) and using the relation \( W_2'(x(w)) = 1/(r + \mu) \), we obtain:

\[
x'(w) = \frac{(r + \mu) U'(w)}{r + \delta + \lambda_1 [1 - F_1(w)]} > 0 \tag{4.4}
\]
The reservation wage is thus unambiguously increasing with $w$.

### 4.2.3 Wage Setting

#### 4.2.3.1 Wage posting in low-productivity jobs

Unemployed workers accept all wage offers above the minimum wage, whereas employees in low productivity jobs accept offers only if these offers exceed their current wage. The unconditional probability that an offer $w$ will be accepted by a randomly contacted worker, represented by $h_1(w)$, is:

$$h_1(w) = \frac{\lambda_0 u + \lambda_1(1-u)G_1(w)}{\lambda_0 u + \lambda_1(1-u)} = \frac{\delta + \lambda_1 G_1(w)}{\delta + \lambda_1}$$

On the other side, a match separation occurs because of a job destruction (at rate $\delta$), because of an external mobility towards other low productivity jobs (at rate $\lambda_1[1 - F_1(w)]$), or because of a promotion towards high productivity jobs (at rate $\lambda_2[1 - \Gamma(\tilde{p}_2(w))]$). Hence, the employer’s value of a continuing match, $J_1(w)$, solves the asset pricing equation:

$$rJ_1(w) = p_1f(k) - w - \{\delta + \lambda_1[1 - F_1(w)] + \lambda_2[1 - \Gamma(\tilde{p}_2(w))}\} J_1(w)$$

Putting it differently,

$$J_1(w) = \frac{p_1f(k) - w}{r + d_1(w)}$$

$$d_1(w) = \delta + \lambda_1[1 - F_1(w)] + \lambda_2[1 - \Gamma(\tilde{p}_2(w))]$$

where $d_1(w)$ denotes the job separation rate.

Note that the job separation rate, $d_1(w)$, is not necessarily decreasing with wage $w$. As rational workers search only for wage increase in the same jobs segment, the probability of finding a better offer from another low productivity job decreases with wage ($\partial \lambda_1[1 - F_1(w)]/\partial w < 0$). However, if the probability of being promoted increases with wage ($\partial \lambda_2[1 - \Gamma(\tilde{p}_2(w))]\}/\partial w > 0$), it could be that $\partial d_1(w)]/\partial w > 0$ at least for some wage rates.

In low-productivity jobs, the wage is posted by the firm. An employer’s expected profit flow per worker contacted is the product of the hire probability per worker contacted and the value of filling a job vacancy net of
4.2. MODEL

investment in human capital. Formally, the wage posting policy of the firm is written as:

\[ w = \arg \{ \max_{w \geq w_h} w \} \]

Let define \( J_1(w) - k = \frac{\kappa(w)}{r + d(w)} \). The first order condition to guarantee the optimality of wage policy is:

\[ \frac{h_1'(w)}{h_1(w)} = \frac{-\kappa'(w)}{\kappa(w)} + \frac{d_1'(w)}{r + d_1(w)} \]

where

\[ \kappa(w) = p_1 f(k) - w - k \{ r + \delta + \lambda_1 [1 - F_1(w)] + \lambda_2 [1 - \Gamma(\tilde{p}_2(w))] \} \]

After some substitutions, the distribution of wage offers in low-productivity jobs segment, \( F_1(w) \), is fully characterized by the following differential equation:\(^1\)

\[ \frac{\lambda_1 G_1'(w)}{\delta + \lambda_1 G_1(w)} + \frac{\lambda_1 F_1'(w) + \lambda_2 \gamma(\tilde{p}_2(w)) \tilde{p}_2'(w)}{r + \delta + \lambda_1 [1 - F_1(w)] + \lambda_2 [1 - \Gamma(\tilde{p}_2(w))]} \]

\[ = \frac{p_1 f(k) - w - k \{ r + \delta + \lambda_1 [1 - F_1(w)] + \lambda_2 [1 - \Gamma(\tilde{p}_2(w))] \}}{p_1 f(k) - w - k \{ r + \delta + \lambda_1 [1 - F_1(w)] + \lambda_2 [1 - \Gamma(\tilde{p}_2(w))] \}} \]

associated with the boundary condition

\[ F_1(w) = 0 \]

4.2.3.2 Wage bargaining in high-productivity jobs

In high-productivity jobs, the wage rate is negotiated between the firm and the worker. Let \( b(p_2 f(k), w) \) denote the negotiated wage conditional on the value of worker product in the high productivity job, \( p_2 f(k) \), and the wage rate of the worker in the low productivity job, \( w \). Because workers in high productivity jobs retire from the market at rate \( \mu \), the value of a high productivity job, denoted by \( J_2 \), solves:

\[ r J_2(b(p_2 f(k), w)) = p_2 f(k) - b(p_2 f(k), w) - \mu J_2(b(p_2 f(k), w)) \]

In high-productivity jobs, the wage rate is negotiated by the firm and the worker after they meet. This view of dual wage determination rules

\(^1\) We also refer ourselves to the envelop theorem to solve the optimal wage policy.
is supported by several empirical studies using French data. Cahuc, Postel-Vinay and Robin [2006] emphasize that low-skilled workers are mainly related to on-the-job search and wage posting, whereas the impact of bargaining power is much more important for high-skilled workers.

Let \( \xi \) represent the bargaining power of the worker. The bargaining game, defined by \( \max_b[W_2(b) - W_1(w)]^\xi[J_2(b) - C]^{1-\xi} \) where \( C \) is the fixed cost of training, yields the following sharing rule:

\[
W_2(b(p_2 f(k), w)) - W_1(w) = \xi [J_2(b(p_2 f(k), w)) - C + W_2(b(p_2 f(k), w)) - W_1(w)]
\]

Let \( \xi \) represent the bargaining power of the worker. The bargaining game, defined by \( \max_b[W_2(b) - W_1(w)]^\xi[J_2(b) - C]^{1-\xi} \) where \( C \) is the fixed cost of training, yields the following sharing rule:

\[
W_2(b(p_2 f(k), w)) - W_1(w) = \xi [J_2(b(p_2 f(k), w)) - C]
\]

By use of the definition of the reservation wage, \( W_1(w) = W_2(x(w)) \), the sharing rule can also be rewritten as:

\[
\frac{b(p_2 f(k), w) - x(w)}{r + \mu} = \left( \frac{\xi}{1-\xi} \right) \left[ \frac{p_2 f(k) - b(p_2 f(k), w)}{r + \mu} - C \right]
\]

Finally, we obtain:

\[
b(p_2 f(k), w) = \xi p_2 f(k) + (1 - \xi)x(w) - \xi(r + \mu)C (4.6)
\]

This equation indicates that the negotiated wage is indeed a weighted average of the value of worker product in a high productivity job and the reservation wage of the worker, net of the discounted cost of training.

### 4.2.4 Firms policies

#### 4.2.4.1 Human capital investment in low-productivity jobs

Optimal investment in human capital, \( k \), maximizes the total expected profit flow, \( \text{i.e.} \)

\[
k = \arg \max \left\{ h_1(w) \left[ \frac{p_1 f(k) - w}{r + d_1(w)} - k \right] \right\}
\]

where \( d_1(w) \) is the job separation rate. The first order condition of optimality requires:

\[
p_1 f'(k) = r + \delta + \lambda_1 [1 - F_1(w)] + \lambda_2 [1 - \Gamma(\bar{p}_2(w))]
\]
so \( k = k(w) \). Namely, the marginal return on employer investment in human capital must equal the discount rate plus the match separation rate. By differentiating the above equation with respect to \( w \), optimal investment policy of the firm is fully characterized by the following differential equation:

\[
k'(w) = \frac{-\lambda_1 F'_1(w) - \lambda_2 \gamma(\tilde{p}_2(w))\tilde{p}_2'(w)}{p_1 f''(k(w))}
\]

(4.7) associated with the boundary condition

\[
k(w) = 0
\]

This boundary condition is a normalization which means that the training amount at the minimum wage is zero.

### 4.2.4.2 Training in high-productivity jobs

Firms also pay a fixed cost of training, denoted by \( C \), for each high-productivity job. The optimal promotion policy of the firm could be summarized by selection of a threshold (reservation) productivity, denoted by \( \tilde{p}_2 \), such that firms could retrieve the cost of training and exploit a positive profit from a new job. Formally, the reservation productivity \( \tilde{p}_2 \) is defined by:

\[
J_2(b(\tilde{p}_2 f(k), w)) = C
\]

Equivalently,

\[
\tilde{p}_2 f(k(w)) = b(\tilde{p}_2 f(k), w) + (r + \mu)C
\]

so \( \tilde{p}_2 = \tilde{p}_2(w) \). Substituting \( b(\tilde{p}_2 f(k), w) \) by (4.6) yields:

\[
\tilde{p}_2(w) f(k(w)) = \xi \tilde{p}_2(w) f(k) + (1 - \xi) x(w) - \xi (r + \mu) C + (r + \mu) C
\]

\[
= x(w) + (r + \mu) C
\]

that is,

\[
b(\tilde{p}_2(w) f(k), w) = x(w)
\]

This equation means that if a worker draws the reservation productivity \( \tilde{p}_2(w) \), he will be promoted but gain zero surplus from the promotion.
Besides, the total surplus associated with this promotion is also zero. Furthermore, we will find:

\[ b(p_2(w)f(k), w) \geq x(w), \ \forall p_2(w) \geq \bar{p}_2(w) \]

Namely, a promotion is always acceptable from the perspective of the worker.

Differentiating the equation \( \bar{p}_2(w) \) with respect to \( w \) gives:

\[ \bar{p}_2'(w) = \frac{x'(w) - \bar{p}_2(w)f'(k(w))k'(w)}{f(k(w))} \quad (4.8) \]

Optimal promotion policy of the firm is then fully characterized by this differential equation and the following boundary condition:

\[ \bar{p}_2(w) = \frac{x(w) + (r + \mu)C}{f(k(w))} \]

4.2.4.3 The signs of \( k'(w) \) and \( \bar{p}_2(w) \)

Now, substituting \( \bar{p}_2(w)' \) by (4.8) into (4.7), we obtain:

\[ k'(w) = \frac{-\lambda_1 F_1'(w) - \lambda_2 \gamma(\bar{p}_2(w))x'(w)}{p_1 f''(k(w)) - \lambda_2 \gamma(\bar{p}_2(w))f'(k(w))\bar{p}_2(w)} > 0 \]

That is, employers who offer higher wages will always invest more in human capital. Note that this result is always true even though the duration of employment is not necessarily monotonously increasing with wage. In the canonical equilibrium search models where firms are allowed to invest in human capital as in Mortensen[1998], the amount of human capital investment is positively correlated with the duration of a job because the probability of external mobility decreases with wage. In this model with two types of job, a raise of human capital also raises the probability that an employee will be promoted, which in turn reduces the expected duration of a job. In equilibrium, the relation between human capital investment and job duration is ambiguous. Consequently, human capital is always increasing with wage whereas the relation between probability of match separation and wage could well be \( U \)-shaped. Further, workers employed at higher wages are more productive even tough all workers are identical ex ante, which in turn results in endogenous within-market productivity differences.
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Next, let’s review equation (4.8) to consider the sign of $\tilde{p}’_2(w)$. A raise of wage $w$ will raise the reservation wage of the worker (first term of the numerator), which raises in turn, by way of wage bargaining, the negotiated wage in a high productivity job. The firm will then respond to that by a raise of the reservation productivity. On the other side, a raise of wage, which goes with an increase of human capital, raises also the value of $f(k)$ in a high productivity job (second term of the numerator). This effect reduces the requirement of the firm in terms of reservation productivity. If the gain linked with increment of match product dominates the loss due to increment of payroll cost, the firm will be less and less exigent in terms of reservation productivity to workers who are richer in human capital. In equilibrium, it could be that the reservation productivity $\tilde{p}_2(w)$ is always decreasing with wage. Consequently, as the probability of being promoted decreases with the requirement of the reservation productivity (measured by $\lambda_2[1 - \Gamma(\tilde{p}_2)]$), lower-paid workers are less likely to be promoted whereas higher-paid workers are more likely to be first promoted. Therefore, the model can predict a positive relation between probability of promotion and wage, along with a $U$-shaped relation between probability of job-to-job transition and wage.

4.2.5 Job-to-job transitions

We decompose total flows of job-to-job transition into two independent components: ascending mobility (promotion) and external mobility (without promotion). Let $m_p(w)$ denote the probability of transition from a low productivity job to a high productivity job (promotion), $m_e(w)$ the probability of external mobility within the same jobs segment, and $m_t(w)$ the total job-to-job transition rate. Formally, these three types of mobility are defined as:

$$m_p(w) = \lambda_2[1 - \Gamma(\tilde{p}_2(w))]$$
$$m_e(w) = \lambda_1[1 - F_1(w)]$$
$$m_t(w) = \lambda_2[1 - \Gamma(\tilde{p}_2(w))] + \lambda_1[1 - F_1(w)]$$
The impact of wage on these transition rates is:

\[ m'_p(w) = -\lambda_2 \gamma(\tilde{p}_2(w))\tilde{p}'_2(w) \]
\[ m'_e(w) = -\lambda_1 F'_1(w) \]
\[ m'_t(w) = m'_p(w) + m'_e(w) \]

Easy to see that if \( \tilde{p}_2'(w) > 0 \), we will always have \( m'_t(w) < 0 \), \( \forall w \). In contrast if \( \tilde{p}_2'(w) < 0 \), a U-shaped relation between probability of job-to-job transition and wage is well allowed as an equilibrium outcome.

### 4.2.6 Labor Market Equilibrium

The appendix B provides a detailed derivation of the labor market equilibrium. Labor market equilibrium in low-productivity jobs segment can actually be summarized by a four-dimensional differential system which jointly defines \{F_1(w), x(x), k(w), \tilde{p}_2(w)\}:

\[
F'_1(w) = \left\{ \frac{1 - k(w)\lambda_2 \gamma(\tilde{p}_2(w))\tilde{p}_2'(w)}{p_1 f(k(w)) - w - k(w) \{ r + \delta + \lambda_1[1 - F_1(w)] + \lambda_2[1 - \Gamma(\tilde{p}_2(w))] \}} \times \right. \\
\left. \frac{\lambda_1}{r + \delta + \lambda_1[1 - F_1(w)] + \lambda_2[1 - \Gamma(\tilde{p}_2(w))] + \lambda_1 \frac{\lambda_1}{r + \delta + \lambda_1[1 - F_1(w)] + \lambda_2[1 - \Gamma(\tilde{p}_2(w))]} \right\}^{-1} \\
x'(w) = \frac{(r + \mu)U'(w)}{r + \delta + \lambda_1[1 - F_1(w)]} \\
k'(w) = -\lambda_1 F'_1(w) - \lambda_2 \gamma(\tilde{p}_2(w))\tilde{p}_2(w)' \\
\tilde{p}_2'(w) = \frac{x'(w) - \tilde{p}_2(w)f'(k(w))k'(w)}{f(k(w))} \]
associated with the following boundary conditions

\[ F_1(w) = 0 \]
\[ x(w) = \left[ \frac{(r + \mu)(r + \lambda_0)}{r(r + \delta + \lambda_0)} \right] \left\{ U(w) + \left( \frac{\delta}{r + \lambda_0} \right) U(z) \right. \]
\[ + \left[ \lambda_1 + \frac{\delta \lambda_0}{r + \lambda_0} \right] \int_w^\infty [W_1(y) - W_1(w)] dF_1(y) \}
\[ k(w) = 0 \]
\[ \tilde{p}_2(w) = \frac{x(w) + (r + \mu)C}{f(k(w))} \]

Given \( F_1(w), x(x), k(w) \) and \( \tilde{p}_2(w) \), it is then straightforward to determine the distributions of wage earnings in low- and high-productivity jobs segment:

\[ G_1'(w) = F'(w) \left[ \frac{\delta + \lambda_1 G_1(w)}{\delta + \lambda_1 [1 - F_1(w)] + \lambda_2 [1 - \Gamma(\tilde{p}_2(w))]} \right] \]
\[ G_2'(b(p_2f(k), w)) = \left( \frac{\lambda_0 \lambda_2}{(\lambda_0 + \delta) n_2 \mu} \right) \left( \frac{G_1'(w) [1 - \Gamma(\tilde{p}_2(w))]}{E [db(p_2f(k), w)/dw]} \right) \]

which satisfy the boundary conditions:

\[ G_1'(w) = 0 \]
\[ G_2'(b(p_2f(k), w)) = 0 \]

where \( n_2 \) solves equation (4.3) and \( b(p_2f(k), w) \) is defined by equation (4.6).

4.3 Estimation

In this section, we proceed to structurally estimate the model. We first provide, in the first subsection, a parametric specification to all the functions invoked to solve the equilibrium. In the second subsection, we present the method of estimation. The third subsection is devoted to present and analyze the estimation results. In the last subsection, we evaluate the empirical performances of the model.

4.3.1 Function specification

Following Rosholm-Svarer[2004], the production function of human capital, \( f(k) \), is specified as follows:

\[ f(k) = (1 + k)^\alpha \]
with the boundary condition:

\[ f(k(w)) = 1 \]

where \( \alpha \in (0, 1) \). This boundary condition means that the value of worker product at the minimum wage is the inherited productivity \( p_1 \).

Next, the utility of incoming \( U(\cdot) \) is specified as a CRRA function:

\[ U(w) = \begin{cases} U_0 + \frac{w^{1-\rho}}{1-\rho} & \text{(if } \rho \neq 1) \\ U_0 + \log(w) & \text{(if } \rho = 1) \end{cases} \]

where \( \rho \geq 0 \). The constant \( U_0 \) is introduced into the utility function to guarantee that the reservation wage \( x(w) \), as well as the negotiated wage \( b(\cdot) \), must be positive in equilibrium.

Finally, we assume that the observed distribution of worker productivity after training for promotion, \( \Gamma(p_2) \), follows a Pareto distribution:

\[ \Gamma(p_2) = 1 - \left( \frac{p_2}{p_2} \right)^\beta \]

where \( \beta > 2 \) and \( p_2 \) is the lower bound of the support, i.e., \( p_2 \geq p_2 \).

4.3.2 Estimation method

We use data from French Labor Survey (FLS 1990-1999), see chapter 2 for presentation of data. The model is evaluated to reflect the market performance of the occupation “skilled manual workers”, as they are particularly related to on-the-job search and envisage the monopsony power of the employer (see Cahuc, Postel-Vinay and Robin [2006]). The following vector (\( \text{dim}(\Phi) = 16 \)) synthesizes all the parameters of the model:

\[ \Phi = \{ r, w, z, \mu, \xi, \delta, \rho, p_1, p_2, \beta, \lambda_0, \lambda_1, \lambda_2, \alpha, C, U_0 \} \]

Given the structure of the model, we turn to the Simulated Method of Moments (SMM) to estimate the model. This method consists in replacing the analytical form of a set of moments restrictions by simulation, then reconciling the set of moments generated by the model with that observed in the data.
3.3. ESTIMATION

Three sets of parameters are separatively considered. The first set, marked with $\Phi_1 = \{r, w, z, \mu, \xi, \lambda_0, \rho\}$ where $\dim(\Phi_1) = 7$, is calibrated based on external information. First, the real interest rate is $r = 4\%$ per annum. The minimum wage rate $w$ and the unemployment compensation $z$ are jointly fixed: minimum wage is normalized to be unity, which corresponds to 1.3 times of average unemployment compensation received by the unemployed workers. The offer arrival rate when unemployed, $\lambda_0$, accounts for the finding that the unemployment spell of skilled manual workers is on average 15 months. It is revealed from the data that skilled manual workers are promoted to intermediate professions in average at 44 years old, that is, they will still work 16 years in the intermediate professions before retirement. Accordingly, the annual probability of retirement is set to $\mu = 0.0541\%$. Existing empirical studies suggest that the bargaining power of the manual workers is low. In line with Cahuc, Postel-Vinay and Robin[2006], we set $\xi = 14\%$ which is an intermediate value over industries. Finally, microdata suggest that the parameter of workers risk-aversion, $\rho = 2$, is an admissible value(see Attanasio-Banks-Meghir-Weber[1999]). All these parameters are reported in table 4.1.

A second set of parameters, marked with $\Phi_2 = \{\lambda_2, C, p_1, p_2, U_0\}$ where $\dim(\Phi_2) = 5$, is calibrated to reproduce some stylized facts:

1. The average yearly promotion rate is 4.5%.

2. The ratio of the average promotion rate of workers employed at wages lied in the 9th decile on the average promotion rate of workers employed at wages lied in the 1st decile, $m_a(D9)/m_a(D1)$, equals 5.9

3. For the occupation “skilled manual workers”, the ratio of the 9th wage decile on the 1st decile, $D9/D1$, equals 1.7

4. For the occupation “intermediate professions”, the ratio of the 9th wage decile on the 1st decile, $D9/D1$, equals 2.1

5. For the occupation “intermediate professions”, the ratio of the 5th wage decile on the 1st decile, $D5/D1$, equals 1.4
These parameters are reported in the table 4.2.

<table>
<thead>
<tr>
<th>$r$</th>
<th>$w_1$</th>
<th>$z$</th>
<th>$\mu$</th>
<th>$\xi$</th>
<th>$\lambda_0$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04</td>
<td>1</td>
<td>0.7692</td>
<td>0.0541</td>
<td>0.14</td>
<td>0.8241</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 4.2: Endogenous Parameter Set $\Phi_2$

<table>
<thead>
<tr>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$\lambda_2$</th>
<th>$U_0$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.21</td>
<td>2.43</td>
<td>0.91</td>
<td>1.95</td>
<td>69.81</td>
</tr>
</tbody>
</table>

We restrict the parameters to be estimated to:

$$\theta = \{\delta, \lambda_1, \beta, \alpha\}$$

where $\dim(\theta) = 4$, $\theta$ denotes the vector of unknown structural parameter. This choice is motivated by the absence of empirical evidence for these crucial parameters of the model. These parameters are evaluated by the SMM estimator. The moments underlying the estimation are based on the wage distribution. We calculate the mean wage within each wage decile, denoted by $w(D_i)$ where $i = 1 \ldots 10$, and the mean wage of the whole sample, denoted by $\tilde{w}$. The subsequent estimations are performed basing on these eleven moments. So formally, the SMM estimator is implemented as follows:

**Step 1** Calculate a 11-dimensional vector of moment, $M_{data}$, from the data.

$$\mathcal{M} = [w(D1), w(D2), ..., w(D10), \tilde{w}]$$

where $D1, \ldots, D10$ denote the wage deciles. This set of moments aims to make a diagnosis whether the model is capable of precisely capturing the shape of actual wage distribution.
4.3. ESTIMATION

Step 2 Given the vector of unknown structural parameter $\theta$, a corresponding set of simulated moments, $M_{model}$, is calculated from the structural model.

Step 3 A SMM estimate $\hat{\theta}_{SMM}$ for $\theta$ minimizes an objective function $Q$ of quadratic form:

$$Q = \min_{\theta} f(\theta) = \min_{\theta} D^T W D$$

where $D = (M_{model} - M_{data})$, $W$ is a symmetric non-negative definite weighting matrix defining the metric\(^2\). Steps 2 and 3 are conducted until convergence, i.e. until an estimate $\hat{\theta}_{SMM}$ that minimizes the objective function is obtained\(^3\).

To examine the empirical performance of the model, we first perform the usual t-tests based on confidence intervals. This sort of hypothesis tests permits to detect whether the vector of unknown parameter, the actual and simulated moments are all precisely evaluated. Next, looking through the moments one by one, we seek to detect how the moments generated by the model coincide with those observed in the data. For any given moment in the vector $D = (M_{model} - M_{data})$, a smaller value indicates that the structural model is able to account for this specific feature of the data, while a larger value may reveal some failures. This leads us to perform a moment-specific diagnostic test. The first order condition associated to the minimization of the objective function $Q$ requires:

$$\left. \left( \frac{\partial D}{\partial \theta} \right) WD \right|_{\theta=\hat{\theta}_{SMM}} = [0]$$

Let $G$ denote the gradient matrix $G = \frac{\partial D}{\partial \theta}$. Using the mean value approxi-

\(^2\)This matrix is given by the inverse of the variance-covariance matrix of the moments obtained from actual data, $W = \left\{ Asy.Cov(\sqrt{N}M_{data}) \right\}^{-1}$ where $N$ is the sample size.

\(^3\)The minimization of the objective function is performed by way of utilities provided by the MATLAB Optimization Toolbox. MATLAB fminsearch finds a minimum of a scalar function of several variables, starting at an initial estimate. This is generally referred to as unconstrained nonlinear optimization.
mation of $G$, one constructs the statistic\footnote{See Collard et al.[2002] for more details.}:

$$T = \left\{ \text{diag}\left[ W^{-1} - G (G'WG)^{-1} G' \right] \right\}^{-1/2} \sqrt{ND}$$

Each element of the vector $T$ follows asymptotically a standard normal distribution $N(0, 1)$.

Following Hansen[1982], we perform finally a global specification test to see whether the set of moments predicted by the model is generally accepted by the data. The related statistic, denoted by $J = ND'WD$, is asymptotically distributed as a chi-square, with a degree of freedom equal to the number of over-identifying restrictions.

\subsection*{4.3.3 Estimation results}

The estimation results are reported in detail in the table 4.3. Giving this table a glance before further discussion, we find that all the unknown structural parameters are precisely estimated(see $t$–stat\textsuperscript{†}). Moreover, the fact that both actual and simulated moments are significantly different from zero makes the set of moments an exigent criterion to test the model’s ability in reproducing actual wage distribution. Looking through the moments one by one, we find that the simulated moments match their empirical counterpart quite well(see $t$–stat\textsuperscript{‡}). So globally speaking, the model allows for a good fit to the actual wage data.

The estimates of two transition parameters, $\delta$ and $\lambda_1$, are comparable to those of Bontemps-Robin-Van den Berg[2000] and Postel-Vinay and Robin[2002] who effectuate the estimation using also French panel data\footnote{Postel-Vinay and Robin[2002] use DADS 1996-1998 panel. Bontemps-Robin-Van den Berg[2000] use FLS 1990-1993 panel. We use FLS 1990-1999 panel.}. With regard to the job destruction rate $\delta$, we find that our estimate is generally the same as those in Postel-Vinay and Robin[2002] and Bontemps-Robin-Van den Berg[2000]: all around 0.07. If we turn to examine the offer arrival rate $\lambda_1$, our estimate $\lambda_1 = 0.40$ constitutes an intermediate value between estimate of Bontemps-Robin-Van den Berg[2000]($\lambda_1$ around 0.1) and that of Postel-Vinay and Robin[2002]($\lambda_1$ larger than 0.6). In general, we
Table 4.3: Estimation Results for SPC “Skilled Manual Worker”

<table>
<thead>
<tr>
<th>Estimates of parameters</th>
<th>$\hat{\theta}_S$</th>
<th>s.d.</th>
<th>$t$ - Stat.</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.9853***</td>
<td>0.0011</td>
<td>881.9836</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>3.6445***</td>
<td>0.1144</td>
<td>31.8452</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.3672***</td>
<td>0.0531</td>
<td>6.9104</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.0684***</td>
<td>0.0101</td>
<td>6.7869</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Estimates of moments</th>
<th>$M_{model}$</th>
<th>$M_{data}$</th>
<th>$t$ - Stat.</th>
<th>Accept.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w(D1)$</td>
<td>1.0791***</td>
<td>1.1248***</td>
<td>1.0242</td>
<td>1</td>
</tr>
<tr>
<td>(0.0446)</td>
<td>(0.0519)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w(D2)$</td>
<td>1.2181***</td>
<td>1.2548***</td>
<td>1.0151</td>
<td>1</td>
</tr>
<tr>
<td>(0.0360)</td>
<td>(0.0291)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w(D3)$</td>
<td>1.3317***</td>
<td>1.3429***</td>
<td>0.3775</td>
<td>1</td>
</tr>
<tr>
<td>(0.0297)</td>
<td>(0.0217)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w(D4)$</td>
<td>1.4265***</td>
<td>1.4209***</td>
<td>0.2224</td>
<td>1</td>
</tr>
<tr>
<td>(0.0252)</td>
<td>(0.0228)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w(D5)$</td>
<td>1.5094***</td>
<td>1.4965***</td>
<td>0.5611</td>
<td>1</td>
</tr>
<tr>
<td>(0.0229)</td>
<td>(0.0215)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w(D6)$</td>
<td>1.5851***</td>
<td>1.5730***</td>
<td>0.5735</td>
<td>1</td>
</tr>
<tr>
<td>(0.0221)</td>
<td>(0.0228)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w(D7)$</td>
<td>1.6596***</td>
<td>1.6566***</td>
<td>0.1339</td>
<td>1</td>
</tr>
<tr>
<td>(0.0223)</td>
<td>(0.0264)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w(D8)$</td>
<td>1.7454***</td>
<td>1.7555***</td>
<td>0.3626</td>
<td>1</td>
</tr>
<tr>
<td>(0.0280)</td>
<td>(0.0314)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w(D9)$</td>
<td>1.8606***</td>
<td>1.8826***</td>
<td>0.5524</td>
<td>1</td>
</tr>
<tr>
<td>(0.0399)</td>
<td>(0.0452)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w(D10)$</td>
<td>2.0457***</td>
<td>2.0692***</td>
<td>0.3252</td>
<td>1</td>
</tr>
<tr>
<td>(0.0724)</td>
<td>(0.0701)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{w}$</td>
<td>1.5451***</td>
<td>1.5568***</td>
<td>0.0412</td>
<td>1</td>
</tr>
<tr>
<td>(0.2839)</td>
<td>(0.2759)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Legend: * Significative at 10% ($t$ - test), ** Significative at 5%, *** Significative at 1%.

$\dagger H_0: \hat{\theta}_{SMM} = 0$ or $M_{model}|data = 0$, $\ddagger H_0: M_{model} = M_{data}$. $Accept. = 1$ if $t - Stat. \ddagger < 1.96$, $Accept. = 0$ otherwise.
find that $\lambda_0 > \lambda_1 > \delta$, a result confirmed in many empirical studies (see also Rosholm-Svarer [2004]).

The estimate of the shape parameter of the function $f(k)$, $\alpha = 0.98$, is relatively high but significantly lower than 1. Finally, the estimate of the shape parameter of the Pareto distribution, $\beta = 3.6$, is a usual value.

### 4.3.4 Discussion

The figure 4.1 gives a comparison between the wage distribution generated by the model and the kernel density estimation of the observed real wages. From the figure, easy to see that the model coincides with the empirical wage distribution generally quite well. Particularly, the predicted wage density overlaps its empirical counterpart on the right side of the wage support. Contrarily, the predicted density fails to capture the empirical shape just at the beginning of the wage distribution (at the lowest wage).

From chapter 3 and this chapter, it is already clear that a bell-shaped wage density will be a typical equilibrium outcome if the flows of promotion are incorporated into the job search model. In most equilibrium search models (including this one), a higher wage will reduce the probability that an employee receives better outside offers in the same jobs segment. This induces a positive relation between the duration of employment and the wage. However, if allow for the possibility of job-to-job transition from low-productivity jobs to high-productivity jobs, a high wage increases also the probability of acceding to high-productivity jobs, i.e., the probability of promotion. This in turn induces a negative relation between the duration of employment and the wage, so represents an opposite force to the positive one. The optimal wage policy of the firm trades off these two competing effects. As we have

---

6 We also calculate the $J - \text{stat}$ of Hansen [1982] as a general test of adequacy. This value is 3.6059 and the associated probability value is 17.61%. Nevertheless, figure 4.1 and table 4.3 clearly indicate that the simulated wage coincides with the actual data quite well both at the global level and moment by moment. Therefore, we could probably have over-evaluated the value of $J - \text{stat}$. Note, particularly, that we use data over a 10-years period. Though the number of observations doesn’t directly affect the value of $J - \text{stat}$ (see appendix C for details), it does indirectly affect the weighting matrix: greater is the number of observations, smaller is the variance-covariance matrix, hence greater the weighting matrix as well as the value of $J - \text{stat}$. We envisage alternative methods to filter the actual wage data and to diminish the variance-covariance matrix.
4.3. **ESTIMATION**

Figure 4.1: Actual and simulated wage distributions

![Wage Distribution](image)

Figure 4.2: Human capital investment in low productivity jobs

![Human Capital Investment](image)
seen, it leads to a market equilibrium where a bell-shaped wage density will be a typical equilibrium outcome.

In figure 4.2, we plot the implied relation between wage and human capital investment, as well as the relation between wage and match product. First, the relation between wages and human capital is increasing and convex. This schema reveals several information: (1) the average amount of investment in human capital is rather low for the whole sample; (2) for low-wage workers, human capital investment is essentially zero; (3) human capital investment is substantial for high-wage workers. For example, the mean wage of the entire sample is 1.5451 (see table 4.3), which corresponds to an amount of human capital investment $k = 0.0385$. However, for a high-wage worker who gains the average wage within the 9th decile, $w(D9) = 1.8606$, the corresponding amount of investment is 0.6326, i.e., approximately 4 months of earnings. The magnitudes of human capital investment are in accordance with Rosholm-Svarer[2004] who, using Danish data, estimate the Mortensen[1998] model and find that “training investments for a person earning 140 DKK (per hour) are approximately 80,000 DKK, which corresponds to approximately 3-4 months of earnings”.

The figure 4.2 also plots the distribution and density of endogenized match product $p_1 f(k)$. Clearly, the density of productivity predicted by the model shows a hump shape which is confirmed by numerous empirical findings using firm data (see for example Bontemps-Robin-Van den Berg[2000]). The figure indicates that worker productivity is rather dispersed in the occupation “skilled manual worker”. The productivity of the highest-paid worker, $p_1 f(\bar{w}) = 9.1289$, is about 4.2 times of the productivity of the least-paid worker. Using FLS data, Bontemps-Robin-Van den Berg[2000] estimate the Burdett-Mortensen[1998] model with continuous productivity dispersion. The authors develop a structural nonparametric estimation method for the productivity distribution. After stratifying the data by industry, the authors find that the estimated productivity is generally distributed from [1, 3] to [1, 6] for any industry in consideration (Bontemps-Robin-Van den Berg[2000], tables 3-4, figures 1-2). Our estimate for the occupation “skilled manual workers” is in accordance with their results.
Figure 4.3: Reservation productivity and match product in high-productivity jobs

NB: $p_2(w)$ represents $\tilde{p}_2(w)$, $p_2(w)f(k) - (r + \mu)C$ represents $\tilde{p}_2(w)f(k) - (r + \mu)C$.

Figure 4.4: Actual and simulated mobilities
The figure 4.3 plots the relation between reservation productivity, \( \tilde{p}_2(w) \), and wage. For wages lower than \( w_1 = 1.6 \) which corresponds to approximately 60th centile of the wage distribution, the requirement of the high productivity firms in terms of reservation productivity remains at a rather high level. In contrast, for wages greater than 60th centile, the requirement of reservation productivity rapidly decreases with wage. Note also that the net match product at reservation productivity, measured by \( \tilde{p}_2(w_1)f(k) - (r + \mu)C \), is always increasing with \( w_1 \). Because the probability of being promoted decreases with the requirement of the reservation productivity, the model predicts a positive relation between probability of promotion and wage.

In figure 4.4, we decompose the total flows of transition into flows of promotion and flows of external mobility, and plot the relation between each type of mobility and wage. Two results are remarkable. First, the predicted probability of promotion is, as expected, increasing with wage. Second, and more importantly, the predicted probability of total job-to-job transition now increases with some high wage rates (from the 8th decile). All these characteristics match well with the observed transition data.

From the figure, the actual relation between probability of promotion and wage is particularly well captured by the model. However, the predicted probabilities of external mobility, as well as the probabilities of job-to-job transition, are generally above their empirical counterparts. Two reasons contribute to explain this result. The first is about the data. As argued in chapter 2, because only one occurrence of job-to-job transition could be identified from one year to another (see section 2.2.4), our measure of job-to-job transitions represents the lower limit of the actual flows in France. Second, most of the empirical studies that have estimated (using French data) different versions of equilibrium search models have problems in fitting actual data of external mobilities. In other words, the estimates of \( \lambda_1 \) are generally too high in this regard, no matter this parameter is estimated by minimizing the discrepancy between the simulated and the actual wage moments (as in this chapter) or by maximizing the likelihood of job duration (as in Postel-Vinay and Robin[2002] or Cahuc, Postel-Vinay and Robin[2006]). In particular,
there are no equilibrium search models that allow for a fit to the French
data of external mobilities associated with wage increases. Indeed, this in-
sufficiency doesn’t root in the models per se, but in the fact that external
mobility is a scare event in France.

4.4 Conclusion

This chapter aims to give a joint explanation of equilibrium wage distrib-
ution and job-to-job transition by emphasizing the role of promotion. We
investigate the impact of the firm behavior to market wage dispersion and, in
particular, promotion. We construct an equilibrium search model with two
types of jobs that differ in the technologies that firms operate. This distinc-
tion refers to differences in occupational categories. The model describes a
two-tier structure of labor market where employees in the upper segment are
stable, but mobile in the lower segment. Firms are allowed to make human
capital investment to low-productivity jobs. Firms also pay a fixed cost of
training for each high-productivity job.

We then estimate the model using data from French Labor Survey(FLS
1990-1999). Generally speaking, the model fits the observed wage data quite
well. In addition, we find that, for high wage workers, the amount of in-
vestment in human capital corresponds to approximately several months (4
months) of wage earnings. However for low wage workers, human capital
investment is essentially zero. The magnitudes of human capital investment
are in accordance with the findings of Rosholm-Svarer[2004]. We also decom-
pose the total flows of transition into flows of promotion and flows of external
mobility. As expected, the predicted probability of promotion is increasing
with wage, at the same time the probability of external mobility is decreasing
with wage. More importantly, the predicted probability of total job-to-job
transition now increases with some high wage rates. All these characteristics
match are consistent with the observed data.

Our explanation of promotion rejoins the explanation stated in Suman[2007]
and Gibbons-Waldman[2003] (in a theoretical framework belonging to the
personal economics). In both papers, the authors all indicate that the prob-
ability of promotion is an increasing function of the wage. The logic behind
the result is essentially the same as we have argued in this chapter: *given past performance, there exists a critical value of current performance above which promotion occurs. And this critical value of current performance above which promotion occurs is lower if past performance has produced a more optimistic belief about innate ability. A wage increase in the past is an indicator about high innate ability in an expected sense and thus the probability of promotion increases in wages (Suman[2007]).* In the two papers like in this chapter, firms always prefer to promote more efficient workers. Worker efficiency is represented, in this chapter, by human capital whereas represented, in Suman[2007] and Gibbons-Waldman[2003], by unobservable ability of the worker. In all three studies, the wage rate serves as an indicator of worker efficiency, the firm then bases their promotion decision on this observed indicator.
Appendix A: Monotonicity of the Negotiated Wage

In this appendix we discuss the monotonicity of the negotiated wage at type-2 jobs. The main argument is that if several exogenous parameters (such as the lowest productivity \(p_2\)) satisfy some sufficient conditions, the expected negotiated wage at a type-2 job is always increasing with the wage at a type-1 job. Consequently, market equilibrium is then always unique.

In the model, the balance of flows into and out of the high-productivity jobs segment is written as:

\[
\begin{align*}
n_2G_2(E[b(p_2 f(k), w)])\mu &= (1 - u)\lambda_2 \int_{w}^{\tilde{p}_2} [1 - \Gamma(\tilde{p}_2(y))]dG_1(y)
\end{align*}
\]

This flow-balance condition holds if the expected negotiated wage, \(E[b(p_2 f(k), w)]\), is strictly increasing with \(w\).

**Proposition 3** (Sufficient condition). The sufficient condition to guarantee that the expected negotiated wage is strictly increasing with wage earned in a low productivity job, \(dE[b(p_2 f(k), w)]/dw > 0, \forall p_2 \in [\tilde{p}_2, \overline{p}_2], \forall w\), is \(\tilde{p}_2'(w) < 0\).

Proof. We rewrite the definition of negotiated wage:

\[
b(p_2 f(k), w) = \xi p_2 f(k) + (1 - \xi) x(w) - \xi (r + \mu) C
\]

Then

\[
E[b(p_2 f(k), w)] = \xi \int_{\tilde{p}_2(w)}^{\overline{p}_2} yd\Gamma(y) f(k) + (1 - \xi) x(w) - \xi (r + \mu) C
\]

\[
\frac{dE[b(p_2 f(k), w)]}{dw} = \xi \left\{ -\tilde{p}_2'(w)\tilde{p}_2(w)\Gamma(\tilde{p}_2(w)) f(k) + \int_{\tilde{p}_2(w)}^{\overline{p}_2} yd\Gamma(y) f'(k)k'(w) \right\}
+ (1 - \xi)x'(w)
\]

Therefore, we will always have \(dE[b(p_2 f(k), w)]/dw > 0\) if \(\tilde{p}_2'(w) < 0\). \(\square\)

**Proposition 4** (Sufficient condition). If \(\overline{p}_2\) is sufficiently high, we will always have \(\tilde{p}_2'(w) < 0, \forall w\).
Proof. We rewrite the definitions of $\tilde{p}_2(w)$:

$$\tilde{p}_2(w) = \frac{x'(w) - \tilde{p}_2(w)f'(k(w))k'(w)}{f(k(w))}$$

Note that the reservation wage $x(w)$ is not affected by $p_2$ (so could be seen as a constant given all the structural parameters of system 1). On the other side, we will always have $\tilde{p}_2 \geq p_2$, $\forall \tilde{p}_2 \in [p_2, \overline{p}_2]$. Therefore, if $p_2$ is sufficiently high, we will always have $\tilde{p}_2'(w) < 0$, $\forall w$. Consequently, we will always have $dE[b(p_2f(k), w)]/dw > 0$, $\forall p_2 \in [\tilde{p}_2, \overline{p}_2], \forall w$.

Economically, the above demonstration transmits the idea that, if the gain linked with increment of match product, $\tilde{p}_2(w)f'(k(w))k'(w)$, dominates the loss due to increment of payroll cost, $x'(w)$, the firm will be less and less exigent in terms of reservation productivity to workers who are richer in human capital. Therefore, the reservation productivity $\tilde{p}_2(w)$ is always decreasing with wage $w$.
Appendix B: Explicit Equilibrium Deduction

Let define $\Sigma(w) \equiv \Gamma(\tilde{p}_2(w))$ to facilitate the solution of system. The labor market equilibrium is characterized by:

$$\Upsilon = \{ F_1(w), G_1(w), \Sigma(w), \tilde{p}_2(w), x(w), k(w), b(p_2 f(k), w), G_2(w), n_2 \}$$

with $\dim(\Upsilon) = 9$. The system is defined on the support $w_1 \in [w_1, w_1]$ and $w_2 = b(p_2 f(k), w)$. The sub-system $\{ F_1(w), \Sigma(w), \tilde{p}_2(w), x(w), k(w), G_1(w) \}$ is calculated by way of ODE (ordinary differential equation) algorithm. Note that the first five differential equations are independent of $G_1(w)$. Known $\{ F_1(w), \Sigma(w), \tilde{p}_2(w), x(w), k(w), G_1(w) \}$, it is straightforward to solve other three functions.

We first write the ODE system:

- $F'_1(w)$

We know

$$\frac{\lambda_1 G'_1(w)}{\delta + \lambda_1 G_1(w)} + \frac{\lambda_1 F'_1(w) + \lambda_2 \gamma(\tilde{p}_2(w))\tilde{p}'_2(w)}{r + \delta + \lambda_1[1 - F_1(w)] + \lambda_2[1 - \Gamma(\tilde{p}_2(w))]} = \frac{1}{k}\left\{ \lambda_1 F'_1(w) + \lambda_2 \gamma(\tilde{p}_2(w))\tilde{p}'_2(w) \right\}$$

Rearranging the terms gives the following expression:

$$F'_1(w) = \left\{ \frac{1 - k(w)\lambda_2 \Sigma'(w)}{p_1 f(k(w)) - w - k(w) \{ r + \delta + \lambda_1[1 - F_1(w)] + \lambda_2[1 - \Sigma(w)] \}} \right\} \times \left\{ \frac{\lambda_1}{\delta + \lambda_1[1 - F_1(w)] + \lambda_2[1 - \Sigma(w)]} + \frac{\lambda_1}{\delta + \lambda_1[1 - F_1(w)] + \lambda_2[1 - \Sigma(w)]} \right\}^{-1}$$
After substituting the production function \( f(k) \) into the above equality, we will have:

\[
F'_1(w) = \left\{ \frac{1 - k(w)\lambda_2\Sigma'(w)}{p_1 [m_0 + (m_1 + k(w))^\alpha] - w - k(w) \{ r + \delta + \lambda_1[1 - F_1(w)] + \lambda_2[1 - \Sigma(w)] \}} \times \right. \\
\left. \frac{\lambda_1}{\lambda_2\Sigma'(w)} \right\} \times \left\{ \frac{\delta + \lambda_1[1 - F_1(w)] + \lambda_2[1 - \Sigma(w)]}{r + \delta + \lambda_1[1 - F_1(w)] + \lambda_2[1 - \Sigma(w)]} \right\}^{-1}
\]

with the boundary condition

\[
F_1(w) = 0
\]

**x'** \( (w) \)

We know that

\[
x'(w) = \frac{(r + \mu)w^{-\rho}}{r + \delta + \lambda_1[1 - F_1(w)]}
\]

with the boundary condition

\[
x(w) = \left[ \frac{(r + \mu)(r + \lambda_0)}{r(r + \delta + \lambda_0)} \right] \left\{ \left( U_0 + \frac{w^{1-\rho}}{1-\rho} \right) + \left( \frac{\delta}{r + \lambda_0} \right) \left( U_0 + \frac{b^{1-\rho}}{1-\rho} \right) \right. \\
+ \left[ \lambda_1 + \frac{\delta\lambda_0}{r + \lambda_0} \right] \int_w^W [W_1(y) - W_1(w)] dF_1(y) \right\}
\]

\[
= \left[ \frac{(r + \mu)(r + \lambda_0)}{r(r + \delta + \lambda_0)} \right] \left\{ \left( U_0 + \frac{w^{1-\rho}}{1-\rho} \right) + \left( \frac{\delta}{r + \lambda_0} \right) \left( U_0 + \frac{b^{1-\rho}}{1-\rho} \right) \right. \\
+ \left[ \lambda_1 + \frac{\delta\lambda_0}{r + \lambda_0} \right] \int_w^W [1 - F_1(y)] W_1'(y) dy \right\}
\]

**k'** \( (w) \)

\[
k'(w) = \frac{-\lambda_1 F_1'(w) - \lambda_2 \Sigma'(w)}{p_1 \alpha(\alpha - 1)[m_1 + k(w)]^{\alpha-2}}
\]

with the boundary condition

\[
k(w) = 0
\]
• $\tilde{p}_2(w)'$

$$\tilde{p}_2(w) = \frac{x'(w) - \tilde{p}_2(w)\alpha[m_1 + k(w)]^{m-1}k'(w)}{m_0 + (m_1 + k(w))}\quad (B-4)$$

with the boundary condition

$$\tilde{p}_2(w) = x(w) + (r + \mu)C$$

• $\Sigma'(w)$

$$\Sigma'(w) = \beta\tilde{p}_2^2(w)^{-\beta-1}\tilde{p}_2(w)'\quad (B-5)$$

with the boundary condition

$$\Sigma(w) = 1 - \left[\frac{p_2}{\tilde{p}_2(w)}\right]^\beta$$

• $G_1'(w)$

$$G_1'(w) = F'(w)\left[\frac{\delta + \lambda_1 G_1(w)}{\delta + \lambda_1[1 - F_1(w)] + \lambda_2[1 - \Sigma(w)]}\right]\quad (B-6)$$

with the boundary condition

$$G_1(w) = 0\quad \square\quad \square\quad \square$$

The explicit expressions of other three functions are as follows:

• $G_2'(b(p_2 f(k), w))$

$$E[G_2'(b(p_2 f(k), w))] = \left(\frac{\lambda_0 \lambda_2}{(\lambda_0 + \delta)n_2\mu}\right) \left(\frac{G_1'(w)[1 - \Gamma(\tilde{p}_2(w))]}{E[db(p_2 f(k), w)/dw]}\right)$$

$$= \left(\frac{\lambda_0 \lambda_2}{(\lambda_0 + \delta)n_2\mu}\right) \left(\frac{G_1'(w)[1 - \Gamma(\tilde{p}_2(w))]}{\xi E(p_2)'f(k(w)) + \xi E(p_2)f'(k(w))k'(w) + (1 - \xi)x'(w)}\right)$$

where

$$E(p_2) = \int_{\tilde{p}_2(w)}^{p_2} p_2 d\Gamma(p_2)$$

$$E(p_2)' = -\tilde{p}_2(w)'\tilde{p}_2(w)\Gamma'(\tilde{p}_2(w))$$
• $b(p_2, w)$

$$b(p_2 f(k), w) = \xi p_2 \left[ m_0 + (m_1 + k(w))^{\alpha} \right] + (1 - \xi) x(w) - \xi (r + \mu) C$$

• $n_2$

$$n_2 = \frac{\lambda_0 \lambda_2}{(\lambda_0 + \delta) \mu} \int_{w}^{\bar{w}} \left[ 1 - \Sigma(y) \right] G_1'(y) dy$$
Appendix C: SMM Estimator—Theory and Implementation

In this section, we briefly present the Generalized Method of Moments (GMM hereunder) estimator and the Simulated Method of Moments (SMM) estimator, their properties and their implementation to estimate the structural model.

C.1 Theory

C.1.1 Introduction to GMM and SMM

As well known, the maximum likelihood (ML) estimator is fully efficient among consistent and asymptotically normally distributed estimators, in the context of the specified parametric model. The possible shortcoming in this result is that to attain that efficiency, it is necessary to make possibly strong, restrictive assumptions about the functional form of the distribution. The generalized method of moments (GMM) estimators move away from parametric assumptions, toward estimators which are robust to some variations in the underlying data generating process.

With regard to the GMM estimator, the estimation of unknown parameters involves optimizing a criterion function based on a set of moment restrictions. Unfortunately, for many econometric models the relevant moment restrictions do not have a tractable analytical form in terms of the unknown parameters rendering the estimation by the generalized method of moments infeasible. In contrast, the simulated method of moments (SMM) estimator modifies the traditional GMM estimator by using moments computed from simulated data of the model rather than the analytical moments. Like the GMM estimator, the SMM estimator is consistent and asymptotically normal when the number of observations tends to infinity, and is asymptotically equivalent to GMM if the number of simulations approaches infinity.
C.1.2 SMM: definition of moments and moment equations

Let $\theta$ be a $k \times 1$ vector of unknown parameter of the structural model. To estimate these parameters, we can construct $q \ (q > k)$ moments—such as mean, variance, etc.—from the structural model. These theoretical moments, denoted by $M_{\text{model}}$, are functions of the unknown structural parameters:

$$M_{\text{model}} = \begin{bmatrix} m_1(\theta) \\ m_2(\theta) \\ \vdots \\ m_q(\theta) \end{bmatrix}_{(q \times 1)}$$

Let also $M_{\text{data}}$ be a $q \times 1$ vector of sample moments deduced from the actual data:

$$M_{\text{data}} = \begin{bmatrix} m_1(\text{data}) \\ m_2(\text{data}) \\ \vdots \\ m_q(\text{data}) \end{bmatrix}_{(q \times 1)}$$

The SMM estimator consist in constructing and solving a system of $q$ moment equations:

$$M_{\text{data}} - M_{\text{model}} = \begin{bmatrix} m_1(\text{data}) - m_1(\theta) \\ m_2(\text{data}) - m_2(\theta) \\ \vdots \\ m_q(\text{data}) - m_q(\theta) \end{bmatrix}_{(q \times 1)} = [0]_{(q \times 1)}$$

The moments will be consistent by virtue of the law of large numbers. They will be asymptotically normally distributed by virtue of the Lindberg-Levy Central Limit Theorem. The derived parameter estimates, denoted by $\hat{\theta}_{\text{SMM}}$, will inherit consistency by virtue of the Slutsky Theorem and asymptotic normality by virtue of the delta method.

C.1.3 Loss function and optimization in SMM

Let $D = M_{\text{data}} - M_{\text{model}}$ be a $q \times 1$ vector which represent the discrepancy between the empirical moments computed from the data and the theoretical
moments obtained from model simulations. The loss function, denoted by $Q$, is written as the weighted quadratic form:

$$Q = D' WD$$

$$= \begin{bmatrix} m_1(data) - m_1(\theta) \\ m_2(data) - m_2(\theta) \\ \vdots \\ m_q(data) - m_q(\theta) \end{bmatrix}_{(1 \times q)} \begin{bmatrix} W_{11} & \cdots & W_{1q} \\ W_{21} & \cdots & W_{2q} \\ \vdots & \vdots & \vdots \\ W_{q1} & \cdots & W_{qq} \end{bmatrix}_{(q \times q)} \begin{bmatrix} m_1(data) - m_1(\theta) \\ m_2(data) - m_2(\theta) \\ \vdots \\ m_q(data) - m_q(\theta) \end{bmatrix}_{(q \times 1)}$$

where $W$ is some $q \times q$ symmetric positive definite weighting matrix. There are $k$ parameters in $\theta$ to estimate, and we have $q$ moment equations. We therefore have $q-k$ overidentifying moment restrictions. If $q=k$ the model is exactly identified, we then return to the Method of Moments (MM) paradigm.

A SMM estimate $\hat{\theta}_{SMM}$ for $\theta$ minimizes the loss function:

$$\hat{\theta}_{SMM} = \arg \min D'WD$$

The first order condition requires:

$$0 = \frac{\partial [D'WD]}{\partial \theta} = 2 \left( \frac{\partial D}{\partial \theta} \right)' WD$$

$$= 2 \begin{bmatrix} \frac{\partial D_1}{\partial \theta_1} & \cdots & \frac{\partial D_1}{\partial \theta_k} \\ \vdots & \ddots & \vdots \\ \frac{\partial D_q}{\partial \theta_1} & \cdots & \frac{\partial D_q}{\partial \theta_k} \end{bmatrix}_{(k \times q)} \begin{bmatrix} W_{11} & \cdots & W_{1q} \\ W_{21} & \cdots & W_{2q} \\ \vdots & \vdots & \vdots \\ W_{q1} & \cdots & W_{qq} \end{bmatrix}_{(q \times q)} \begin{bmatrix} D_1 \\ \vdots \\ D_q \end{bmatrix}_{(q \times 1)}$$

### C.1.4 Asymptotic Properties of SMM

GMM estimates are typically consistent (by virtue of laws of large numbers and Slutsky’s theorem) and asymptotically normally distributed (by virtue of central limit theorems). Formally, let $n$ denote the sample size, $\theta_0$ denote the true parameter vector, and $G = p \lim \frac{\partial D(\theta)}{\partial \theta} = \frac{\partial D(\theta_0)}{\partial \theta_0}$ denote the asymptotic gradient matrix of the discrepancy of empirical and theoretical moments with respect to the parameters. Then we have:

$$\hat{\theta}_{SMM} \xrightarrow{p} \theta_0$$

$$\sqrt{n}(\hat{\theta}_{SMM} - \theta_0) \xrightarrow{d} N \left( \begin{bmatrix} 0 \end{bmatrix}_{(k \times 1)}, \left[ G'WG \right]^{-1} \right)$$
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The first relation states that $\hat{\theta}_{SMM}$ converges in probability to $\theta_0$, the second states that the limiting distribution of $\hat{\theta}_{SMM}$ is standard normal.

C.1.5 Efficient Weighting Matrix

Following Greene[2002], the weighting matrix $W$ is given by the inverse of the covariance matrix of $\sqrt{n}$ times the empirical moments obtained from actual data:

$$W = \{Cov(\sqrt{n}M_{data})\}^{-1} = \{nCov(M_{data})\}^{-1}$$

Note that the above weighting matrix $W$ depends only on data but not on model (as well as the parameters).

C.1.6 Tests

C.1.6.1 Over-identification Test (J-test)

Because we have $k$ unknown structural parameters and $q$ moment restrictions, we have therefore $q - k$ remaining overidentifying restrictions which should also be close to zero if the model is correct (fits data). Following Hansen[1982], we perform finally a global specification test to see whether the set of moments predicted by the model is generally accepted by the data. The related statistic, denoted by:

$$J = nD'WD = [\sqrt{n}D'] \{Cov(\sqrt{n}M_{data})\}^{-1} [\sqrt{n}D]$$

is asymptotically distributed as a chi-square, with a degree of freedom equal to the number of over-identifying restrictions:

$$J \xrightarrow{d} \chi^2_{q-k}$$

For the exactly identified case, there are zero degrees of freedom $q - k = 0$, and consequently $J = 0$.

C.1.6.2 Tests based on confidence intervals (Asymptotic t-test)

The confidence interval gives a range of plausible values for the parameter. Therefore, it stands to reason that if a hypothesized value of the parameter
does not fall in this range of plausible values, then the data are not consistent with the hypothesis, and it should be rejected. Consider testing

$$H_0 : \theta = \theta_0$$

$$H_1 : \theta \neq \theta_0$$

Then, $H_0$ is rejected if

$$\left| \frac{\theta - \theta_0}{s.d.(\theta)} \right| > C_{1-\alpha/2}$$

where $\alpha$ is the significant level, and $C_{\alpha/2}$ denotes the critical value of a two-tails distribution at a significant level of $1 - \alpha/2$. Because the moments and the auxiliary parameters are normally distributed, this critical value is 1.96 for $\alpha = 0.05$.

This sort of hypothesis tests is usually used to detect whether or not the following assumptions hold:

$$H_0 : \hat{\theta}_{SMM} = 0$$

$$H_0 : M_{model} = 0$$

$$H_0 : M_{data} = 0$$

$$H_0 : M_{model} = M_{data}$$

**C.1.6.3 Moment-specific diagnostic test (T-test)**

Beyond these traditional statistical tests, we use also a diagnostic test in line with Collard et al.[2002], that locates the potential failures of the structural model. The first order condition associated to the minimization of the objective function $Q$ requires:

$$\left( \frac{\partial D}{\partial \theta} \right)' WD \bigg|_{\theta = \hat{\theta}_{SMM}} = [0]$$

Using the mean value approximation of the gradient matrix $G$, one constructs the statistic:

$$T = \left\{ \text{diag} \left[ W^{-1} - G (G'WG)^{-1} G' \right] \right\}^{-1/2} \sqrt{nD}$$

where $n$ is the sample size. Each element of the vector $T$ follows asymptotically a standard normal distribution.
C.2 Implementation

The SMM estimator is implemented in MATLAB as follows:

Step 1 Calculate a $q$-dimensional vector of moment, $M_{data}$, from the data. Import the covariance matrix of the empirical moments, $W$.

Step 2 Given the vector of unknown structural parameter, $\theta$, a corresponding set of simulated moments, $M_{model}$, is calculated from the structural model.

Step 3 A SMM estimate $\hat{\theta}_{SMM}$ for $\theta$ minimizes the loss function $Q$.

Steps 2 and 3 are conducted until convergence, i.e. until an estimate $\hat{\theta}_{SMM}$ that minimizes the objective function is obtained.

Step 4 Perform post-estimation tests.

C.2.1 Importing the covariance matrix

For our model, we need to calculate the covariance matrix for the vector of wage and the ten vectors of wage deciles:

$$cov([\tilde{w}, w(D1), w(D2), \cdots, w(D10)])$$

In the data, we have only a vector of wage $\tilde{w}$. The ten vectors of wage deciles are constructed by:

$$w(D1) = \tilde{w} \text{ if } \tilde{w} \in [w, D1]$$
$$w(D2) = \tilde{w} \text{ if } \tilde{w} \in (D1, D2]$$
$$\vdots$$
$$w(D10) = \tilde{w} \text{ if } \tilde{w} \in (D9, w]$$

Then the covariance matrix could be calculated in any statistical software such as STATA, SAS, or others.

The covariance matrix being imported into MATLAB, the weighting matrix, $W$, is calculated as:

$$W = \{n \times cov ([\tilde{w}, w(D1), w(D2), \cdots, w(D10)])\}^{-1}$$
C.2.2 Minimizing the loss function

The minimization of the objective function is performed by way of utilities provided by the MATLAB Optimization Toolbox. MATLAB fminsearch finds a minimum of a scalar function of several variables, starting at an initial estimate. This is generally referred to as unconstrained nonlinear optimization.


C.2.3 Calculating the probability associated with a normally-distributed statistic

Once SMM estimation is achieved, we need to perform some post-estimation tests such as $J$-test, $t$-test or $T$-test.

Let’s first consider the $t$-test. The related statistic, $t$-statistic, follows a standard normal distribution. Now we seek to calculate the probability that the null assumption $H_0$ holds. This probability, denoted by $p$, is:

$$\text{Prob}(|z| > t > 0) = p \iff 1 - P(|z| \leq t) = p$$

$$\iff 1 - [\Phi_0(t) - (1 - \Phi_0(t))] = p$$

$$\iff 2[1 - \Phi_0(t)] = p$$

Note that the same logic applies to the $T$-statistic because this statistic also follows a standard normal distribution.

C.2.4 Calculating the probability associated with a chi-square-distributed statistic

Now consider the case of $J$-statistic. This statistic is asymptotically distributed as a chi-square, with a degree of freedom equal to the number of over-identifying restrictions, $J \rightarrow \chi^2_{q-k}$. The probability that the model is over-identified, denoted by $p$, is:

$$p = F(J|J \rightarrow \chi^2_{q-k}) = \int_0^J \frac{t^{(q-k-2)/2}e^{-t/2}}{2^{(q-k)/2}\Gamma((q-k)/2)}dt$$

where $\Gamma(\cdot)$ is the Gamma function. Naturally, this probability is increasing with the value of $J$-statistic.
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General Conclusion

In this thesis, we first survey the recent developments of equilibrium search theory and focus on a fundamental question of labor economics: why are workers paid differently? Both empirical studies and theoretical investigations indicate that actual wage differentials are mainly explained by firm heterogeneity (measured and unmeasured) in interaction with market frictions. Note, in particular, that wage dispersion will vanish if market is frictionless or if all firms are identical (in both measured and unmeasured characteristics). Therefore, it is rather the interaction between firm heterogeneity and market frictions that provides a good fit to within-market wage distributions.

We then proceed to empirical analysis of equilibrium search model along two directions: not only wage dispersion but also job-to-job transitions. Using French panel data (French Labor Survey 1990-1999), we decompose the total flows of job-to-job transition into flows of promotion and flows of external mobility. Formally, we consider three types of job-to-job mobility: external mobility (without promotion), external promotion and internal promotion. By promotion, we mean a raise of occupational category of the employee in sense of Social Professional Category (SPC). By external mobility, we mean a job-to-job mobility via a change of firm (made with an episode of unemployment of one month or less). Importantly, an internal promotion should also be considered as a job-to-job transition because it concerns a change of job (see Lazear-Oyer[2004]).

Our empirical investigations lead to three basic conclusions. First, in the decade 1990-1999, around 40% to 55% of job-to-job transitions result in lower wages in new jobs. However, the literature is just beginning to develop models that have as a feature job changes with negative wage changes. Second, in this
decade, around two thirds of job-to-job transitions are in nature promotion in sense of a raise of occupational category (SPC). Further, around 90% of promotions are made within firms, whereas only 10% are made via a change of firm. Therefore, internal promotions account for around 60% of total job-to-job transitions. Nevertheless, most equilibrium search models don’t allow to deal with this particular type of professional mobility. Third, the frequency of external mobilities is unambiguously decreasing with wage earnings for any SPC in consideration. This empirical evidence confirms the prediction of equilibrium search theory that external mobility is a decreasing function of wage. On the other side, the frequency of promotions is always increasing with wage earnings. In other words, a higher-wage worker is more likely to be promoted than a lower-wage worker. So globally, the relation between frequency of job-to-job transitions and wage earnings is $U$-shaped, typically first decreasing then increasing.

From a theoretical standpoint, we point out that the flow of job-to-job transition from one occupational category (SPC) to another is a dimension up to now largely neglected. Importantly, we argue that extending the equilibrium search model to account for this dimension is a fruitful way to revisit the shape of equilibrium wage distribution.

Worker behavior and firm behavior are both important determinants of job-to-job transition and, in particular, promotion. Indeed, the Burdett-Mortensen[1998] model shows that external mobility principally results from on-the-job search behavior of the employee in response to monopsony market power and wage posting policy of the firm. Nevertheless, this argument suitable to explain external mobility could not be applied to explain worker promotion. In this thesis, we propose two explanations of worker promotion. From one side, the employee could search on-the-job for promotion. From the other side, the firm could control the probability of promotion of the employee. Both arguments help to explain actual wage dispersion. Importantly, it is revealed that firm effect has much more significant impact on worker promotion than person effect. The underlying reason is that, unlike external mobility, worker promotion is mainly a result of firm decision rather than endogenized search behavior of the worker. Therefore, any model that explains worker promotion from the side of the firm will provide better co-
Incidence with the actual data of promotion, as well as job-to-job transition.

In this thesis, we develop the equilibrium search theory to give a joint explanation of wage distribution and labor market transition, by emphasizing the role of promotion. Nevertheless, we also envisage several extensions of interest that deserve further concern and research. We list below two desirable extensions.

The first extension goes into the direction of analyzing the determinants of external mobilities with wage decreases. In this regard, a further empirical investigation using linked employee-employer data is very desirable. The literature is just beginning to develop models that have as a feature job changes with negative wage changes. Examples of models that allow to explain this particular type of mobility include Jolivet, Postel-Vinay and Robin[2002] who emphasize exogenous job reallocation shock; Postel-Vinay and Robin[2002] who emphasize that workers accept a lower starting wage via job-to-job transition initially because they envisage the potential of within-firm wage growth in the future(see also Connelly-Gottschalk[2002]); Bowlus-Villuber[2001] who emphasize that workers lower their reservation wage below their current wage due to their impending displacement; and Horny, Mendes and Van den Berg[2006], Dey-Flinn[2003], Gorgens[2002] and Sullivan[2003] who emphasize other job attributes. However, whether and to what extent these models allow to explain and capture the actual relation between this particular type of mobility and the wage remains an open question.

Another extension consists in investigating the relation between probability of promotion within-firm and wage. Throughout this thesis, the promotion is defined as a raise of occupational category of the employee in sense of Social Professional Category (SPC). However, there are also other ways to define promotion. Importantly, promotion inside the firm is typically regarded as a raise to a higher job level. The theoretical work of Suman[2007] and Gibbons-Waldman[2003] indicate that the probability of promotion should be an increasing function of the wage. The logic behind the result is essentially the same as we have argued in chapter 4: firms always prefer to promote more efficient workers. The wage rate serves as an indicator of worker efficiency, the firm then bases their promotion decision
on this observed indicator. Therefore, an in-depth empirical investigation of this relation is also interesting, and constitutes another direction of future research.
Bibliography


[21] O.J. Blanchard and C.A. Diamond. The flows approach to labor mark-

equilibrium job search model with search on the job and heteroge-
75, 1999.

with continuous productivity dispersion: Theory and non-parametric

wage distributions with heterogeneity. *Journal of Applied Economet-

models and the transition from school to work. *International Economic


[27] A.J. Bowlus and L. Vilhuber. Displaced workers, early leavers and

of Essex, Colchester.*, 1990.

[29] K. Burdett and M.G. Coles. Marriage and class. *Quarterly Journal of


[31] K. Burdett and M.G. Coles. Equilibrium wage-tenure contracts. *Econo-

50:955–70, 1983.


